OFDM & OFDMA

Orthogonal Frequency Divison Multiplexing and Orthogonal Frequency Division Multiple Access

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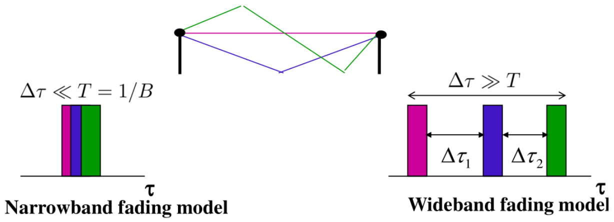


Fig. 1. Fading Models

***Abstract*—As mobile wireless systems around the world evolve, the underlying technology is changing from ones based on direct-sequence spread spectrum (DSSS) to ones based on orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA). Third-generation (3G) systems are based mostly on DSSS, such as Evolution-Data Optimized (EV-DO) and High-Speed Packet Access (HSPA). Most fourth-generation (4G) systems use OFDM and OFDMA, including Mobile WiMAX and Long Term Evolution (LTE).**

This evolution in wireless wide area networks (WWANs) is not surprising given that we have already seen a similar shift in wireless local area networks (WLANs). Earlier, lower-speed versions of the IEEE 802.11 standards, such as IEEE 802.11b, use DSSS at the physical layer. Later, higher-speed versions, such as IEEE 802.11g and IEEE 802.11n, predominantly use OFDM. One reason for such a shift is that OFDM offers some intrinsic advantages in delivering high-speed data, especially in a multipath, frequency-selective fading environment.

I. MOTIVATION FOR MULTIPLE CARRIER SUBSYSTEMS

A. Narrow Band Fading Model

As we can see in Figure(1), for narrowband fading model, $\Delta\tau << T$. Hence its not a problem when received at output.Here multipath components are non-resolvable.

B. WideBand Fading Models

In this model, individual multipath components are resolvable. In the time domain, multipath leads to the **spreading out** of the arrival time of received signals due to multiple propagation paths through which signals travel. This dispersion of arrival time is called channel delay spread τ.

In a high-speed wireless system, the symbol rate R_s is high, hence the symbol time T_s is low. As the symbol rate R_s becomes higher and the symbol time T_s becomes shorter, eventually T_s can become much shorter than the channel delay spread τ (i.e., $T_s << \tau$) for a given channel. When the symbol time becomes small as compared to the channel delay spread, the delayed versions of one symbol start to leak into and interfere with the subsequent symbol (see Figure 2). This phenomenon, called ISI, turns out to be a major impediment to high-speed wireless systems.

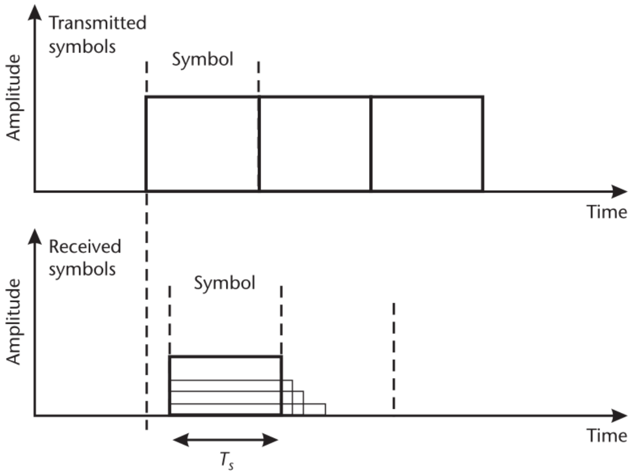
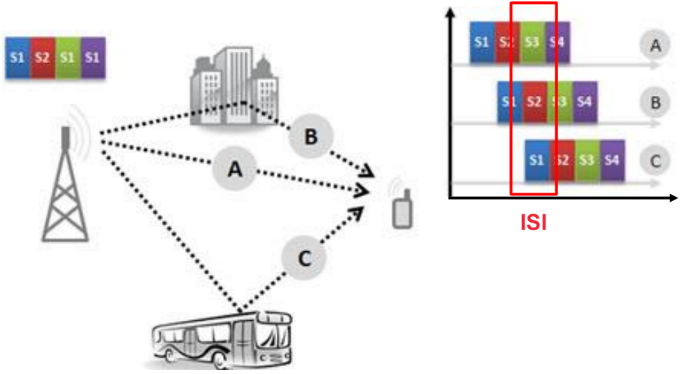


Fig. 2. Intersymbol Interference(ISI)



C. Solution for Inter Symbol Interference(ISI)

The way to address the ISI problem lies in choosing narrowband subcarriers for transmission. Transmitting narrowband subcarriers addresses the problem of ISI by artificially lengthening the symbol time. The symbol time is lengthened by reducing the symbol rate, and the symbol rate is reduced by dividing the high-rate symbol stream into many low-rate symbol substreams, each with a lower symbol rate R_s/K (K is number of subcarriers).

As a result, the symbol time of each low-rate symbol substream becomes T_sK. Thus, from the perspective of each low-rate substream of data symbols, each symbol experiences little ISI. This is so because when the symbol time of a symbol in the substream becomes large relative to the channel delay spread τ (i.e., $T_sK >> \tau$), delayed versions of a symbol have little effect on the next symbol.

In the **frequency domain**, multipath leads to “nulls” in the frequency response of the channel in frequency . Thus, multipath fading is also known as frequency- selective fading. A frequency-selective channel is characterized by the coherence bandwidth W_c , which is the bandwidth over which the channel appears relatively flat and unvarying.

Transmitting narrowband subcarriers addresses the problem of frequency-selective fading by artificially dividing a wideband carrier into smaller narrowband subcarriers. As a result, each narrowband subcarrier has a much smaller bandwidth (than the original wideband carrier). Each narrowband subcarrier undergoes its own fade. If the bandwidth of each narrowband subcarrier is sufficiently small (usually much smaller than the coherence bandwidth), then each narrowband sub-carrier can be said to undergo flat fading. Thus, from the perspective of each narrowband subcarrier, the subcarrier experiences little frequency-selective fading(see Figure 3). In other words, when the bandwidth R_s/K of a subcarrier becomes small relative to the channel coherence bandwidth W_c (i.e., $R_s/K << W_c$), frequency-selective fading is substantially reduced.

As there are narrowband sub-carriers, modulation techniques and coding per subcarrier can be incorparated for better performance. When the system puts data symbols across multiple subcarriers all over the band, at any given time some subcarriers experience fades but other subcarriers experience no fades. For those subcarriers that experience fades, they can fall back to a more robust modulation (e.g., quadrature phase shift keying or QPSK) and/or lower-rate error correction code (e.g., rate 1/3 convolutional code). Doing so increases the chance that data symbols will be received without errors but reduces the effective bit rate. For those subcarriers that experience little fades, they can take advantage of a more efficient modulation scheme (e.g., 16-quadrature amplitude modulation or 16-QAM) and/or higher-rate error correction code (e.g., rate 3/4 convolutional code). Doing so increases the effective bit rate without sacrificing the error rate. By **adapting modulation** and **coding for each subcarrier**, the system can achieve the best possible overall capacity and performance.

The receiver needs an equalization function to invert (i.e., equalize) the channel response.When a subcarrier is narrow, the required equalization function for that subcarrier in the receiver is simpler. This is because a narrow subcarrier in frequency means a long transmission symbol in time. Note that in a receiver, a channel equalizer is still required for each subcarrier. Thus a total of K (albeit simpler) equalizers are needed in the receiver.

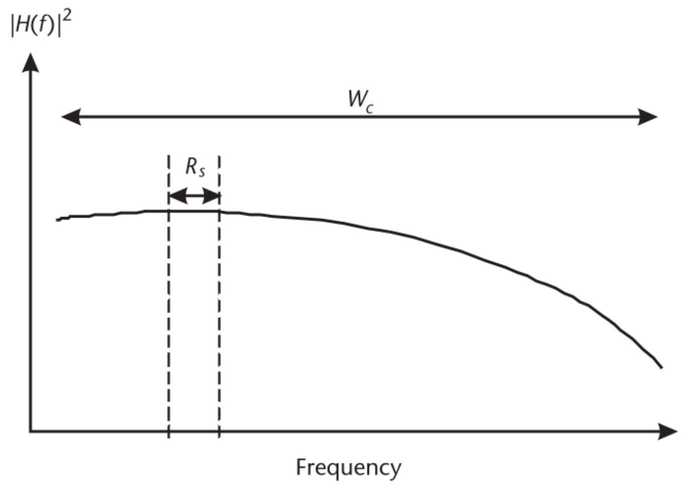


Fig. 3. By keeping the bandwidth of a subcarrier much less than the coherence bandwidth, the subcarrier is experiencing an approximately flat channel.

The robustness against ISI and multipath fading is the key advantage of using multiple narrowband subcarriers, and this advantage is carried into FDM,FDMA,OFDM and OFDMA.

II. CONVENTIONAL FDM

Frequency-division multiplexing (FDM) is an analog technique that can be applied when the bandwidth of a link (in hertz) is greater than the combined bandwidths of the signals to be transmitted

Figure(4) shows the transmitter portion of a conventional digital FDM system. At the input of the transmitter, there is a single high-rate stream of baseband data symbols running at a rate of R_s symbols per second (sps). This high-rate stream of baseband data symbols consists of blocks of complex data symbols, and each block contains L complex data symbols. A serial-to-parallel (S-to-P) converter converts the high-rate stream into K separate low-rate substreams. As a result, each low-rate substream has a rate of R_s/K sps. Also, the serial-to-parallel converter breaks the one large block containing L symbols into K smaller blocks in parallel, each containing L/K data symbols. Each low-rate substream goes through a digital-to-analog (D-to-A) converter and is then modulated by its own complex sinusoid $\exp(-j2\pi f_k t)$, where f_k is the subcarrier frequency assigned to each low-rate substream. After modulation, the K modulated subcarriers at K different frequencies are summed, and the composite signal is then transmitted over the air.

Figure(5) depicts the spectrums of the high-rate stream and the K low-rate substreams. The spectrums shown are for the continuous-time equivalents of the symbol streams. While we do not go into details of computing the spectrums, it suffices for our purposes to state that the bandwidth of a stream of symbols is mostly confined to $1/T_s$, where T_s is the symbol time (duration) of a symbol in the stream. Since the symbol

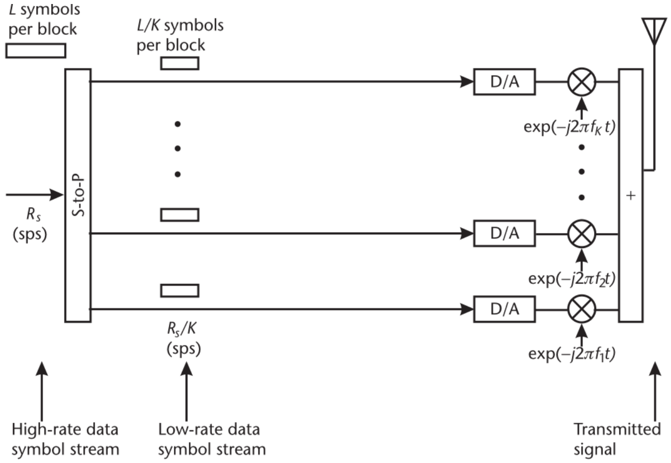


Fig. 4. A conventional FDM transmitter

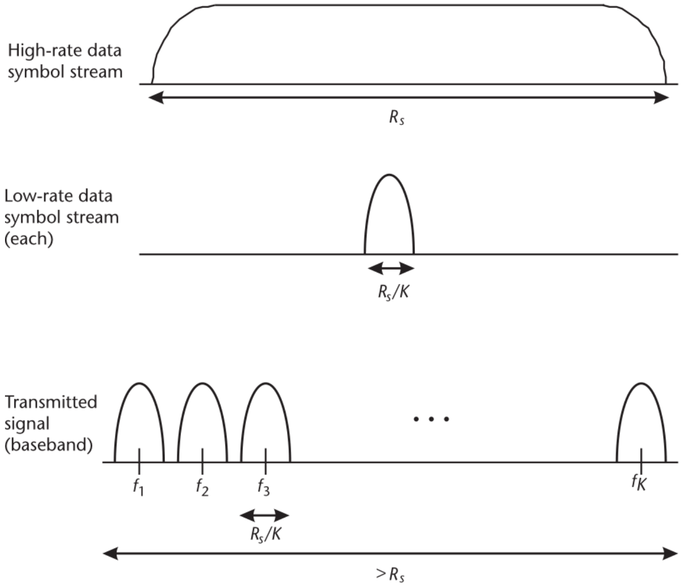


Fig. 5. The spectrums of the high-rate data symbol stream, the low-rate data symbol sub- streams, and the transmitted FDM signal.

rate $R_s = 1/T_s$, for now we can assume that the bandwidth of a stream of symbols is limited to its symbol rate R_s.

In this example, the high-rate stream has a symbol rate of R_s sps and a spectrum that is R_s Hz wide; each symbol lasts $1/R_s$ s. The serial-to-parallel converter divides the high-rate stream into K low-rate substreams, and each low-rate sub-stream now has a lower rate of R_s /K sps and a narrower spectrum that is R_s /K Hz wide. Because of the lower rate, each symbol in the substream lengthens to K/R_s s. Then, each low-rate substream is modulated by a complex sinusoid $\exp(-j2\pi f_k t)$ to a different frequency f_k . After summation, the composite signal consists of K signals multiplexed in the frequency domain. As a result, K separate low-rate, narrowband subcarriers are used to transmit the original high-rate, wideband stream.

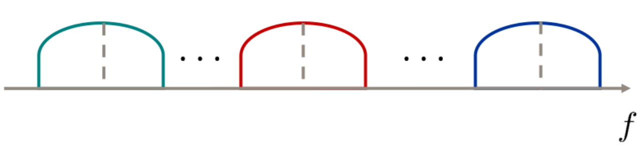


Fig. 6. Guard bands for FDM

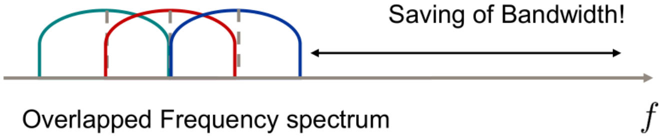


Fig. 7. Overlapping frequency spectrum of OFDM

To minimize interference between subcarriers, a guard band is placed between two adjacent subcarriers. In Figure 4, the K subcarriers have frequencies $f_1, f_2, \ldots f_K$ that are sufficiently spaced to minimize interference between subcarriers. Because of the guard bands between subcarriers, the total bandwidth occupied by the K subcarriers is greater than simply K times the bandwidth of each subcarrier. Although Figure(5) shows that there are K different subcarriers, in FDM all K subcarriers are used to carry data for one user only.

A. Advantages of FDM

- It is effective at combating intersymbol interference (ISI) and multipath fading;
- It can adjust modulation and coding for each subcarrier; and
- It has simple equalization.

B. Disadvantages of FDM

- The transmitter needs to have K separate D-to-A converters and K separate radio frequency (RF) modulators.
- FDM is not bandwidth efficient. The extra guard bands necessarily add to the total bandwidth requirement.(see Figure 6).

So is there a way to address these disadvantages and, at the same time, retain the advantage of transmitting multiple narrowband subcarriers? The answer is yes - in the form of **OFDM** (Figure 7).

III. OFDM

A. Basics of OFDM

In OFDM, the objective is still to transmit a high-rate stream using multiple subcarriers. OFDM overcomes the problem of the large bandwidth requirement imposed by guard bands. Instead of using K local oscillators (LOs) and K multipliers in modulation, OFDM uses a mathematical technique called **discrete Fourier transform (DFT)** to generate the subcarriers. The subcarriers generated this way do not need additional guard bands and can be placed closer together in the frequency domain. The subcarriers are also orthogonal to each other over

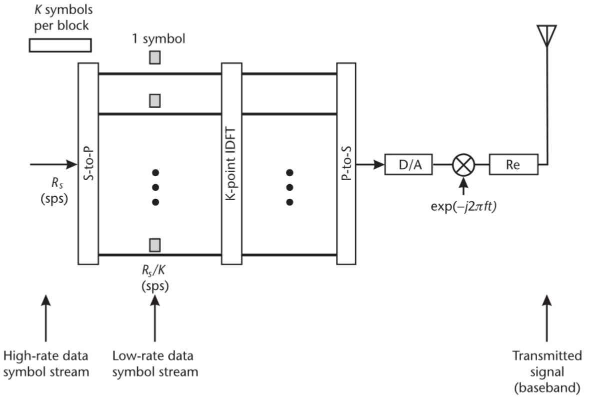


Fig. 8. An OFDM transmitter

a set duration (i.e., over the duration of an OFDM symbol). In addition, DFT and its inverse can be efficiently computed, eliminating the need for separate RF components for separate subcarriers.

Figure(8) depicts a simplified OFDM transmitter matching the example presented in the FDM section. The high-rate stream of data symbols is still running at a rate of R_s sps, and each data symbol lasts $1/R_s$ s. This high-rate stream of data symbols consists of blocks of complex data symbols, and each block contains K complex data symbols.

Since K subcarriers are to be generated, the serial-to-parallel converter converts the high-rate stream into K separate low-rate substreams; each low-rate substream has a rate of $R_s/Ksps$. In doing so, the serial-to-parallel converter assigns successive data symbols (at its input) to K separate substreams (at its outputs). So at any given time at the output of the serial-to-parallel converter, there is a set of K data symbols in parallel.

The set of K data symbols in parallel pass through the inverse DFT (IDFT) function, which transforms the K data symbols. After IDFT, the K transformed symbols in the K substreams then pass through the parallel-to-serial (P-to-S) converter that puts the K transformed symbols in series. This block of K transformed symbols in series constitutes a single block or an **OFDM symbol**. Successive OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K OFDM symbols per second, and each OFDM symbol lasts K/R_ss. Note that an OFDM symbol is different from a data symbol, which encodes one or more user bits and is the input to the serial-to-parallel converter.

Figure 9 shows the spectrums of the high-rate stream, the low-rate substreams, and the transmitted signal. In particular, the spectrum of the transmitted OFDM signal is shown over one block or one OFDM symbol. In the transmitted OFDM signal, the subcarriers are separated such that they physically overlap in frequency, but the first zero crossings of one subcarrier fall on the peaks of the two adjacent subcarriers. In fact, all zero crossings of a subcarrier fall on the peaks of all adjacent subcarriers. Because OFDM recovers the data symbol at the peak of each subcarrier, the subcarriers are orthogonal to each other and there is no interference — hence the term orthogonal FDM (OFDM).

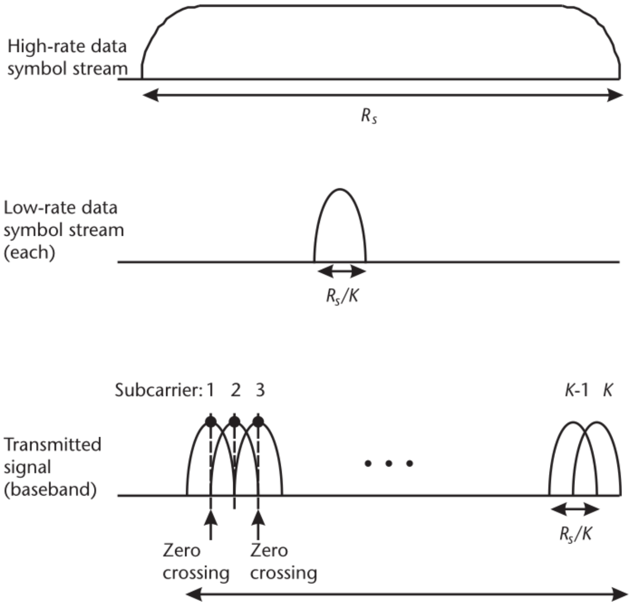


Fig. 9. The spectrums of the high-rate data symbol stream, the low-rate data symbol sub- streams, and the transmitted OFDM signal. The spectrum of the transmitted OFDM signal is shown for the duration of one OFDM symbol. Note that the first zero crossings of subcarrier 2 fall on the peaks of adjacent subcarriers 1 and 3.

Although Figure(9) shows that there are K different subcarriers, all K subcarriers in a block (an OFDM symbol) are assigned to only one user. In other words, only one user transmits in a block (an OFDM symbol).

B. Cyclic Prefix

The IDFT function transforms the set of K parallel data symbols from the frequency domain into the time domain. In OFDM, the system pretends that the data symbols originally exist in the frequency domain. That is why later at the receiver,the data symbols are recovered at the peaks of the (overlapping) sync functions in the frequency domain. In any case, the set of K transformed symbols in parallel then pass through the parallel-to-serial converter, which puts the K transformed symbols in series. A set of K transformed symbols in series is called an OFDM symbol, and the OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K (OFDM symbols per second or blocks per second). Then the OFDM symbols are upconverted to produce the transmitted signal.

The simplified transmitter just presented has a problem. Figure 10 dictates that each OFDM symbol comes right after the previous OFDM symbol, without any guard time in between. Figure 11 shows this serial stream of OFDM symbols (each OFDM symbol contains K transformed symbols) with no guard time. Because of the multipath, delayed versions of an OFDM symbol can fall on the next OFDM symbol. As a re-

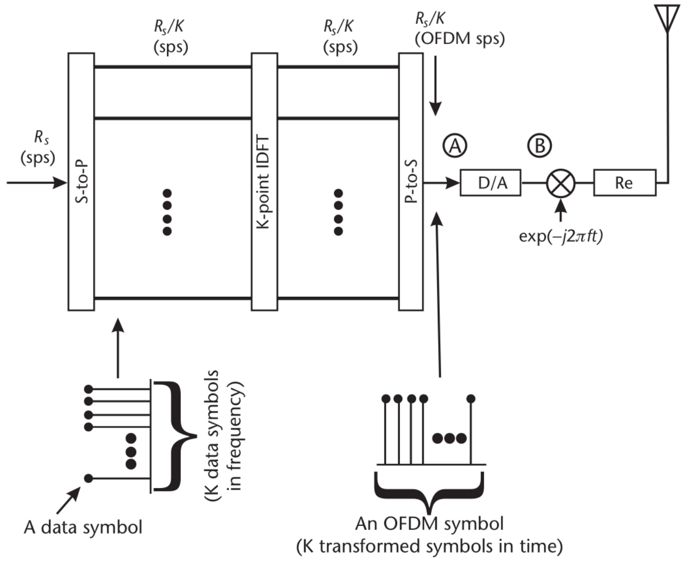


Fig. 10. An OFDM transmitter

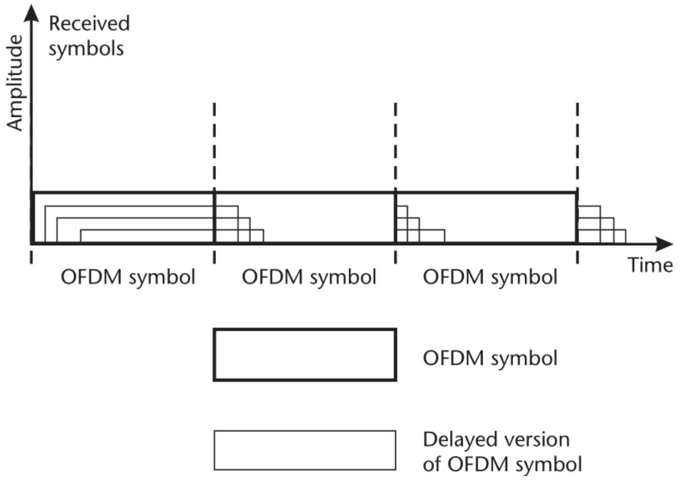


Fig. 11. OFDM symbols without guard time and with IBI.

sult, there is inter-OFDM symbol interference (i.e., interblock interference, or IBI) between adjacent OFDM symbols.

An advantage of OFDM is that the data symbols in a single OFDM symbol do not interfere with one another inside an OFDM symbol. To state it in another way, the K data symbols do not affect one another within the symbol time of an OFDM symbol as far as data recovery at the receiver is concerned. This is because data are recovered at the peaks of the overlapping sync functions (in the frequency domain).

To reduce such interference between adjacent OFDM symbols, one needs to add an extra guard time between adjacent OFDM symbols. In practice, extra symbols are inserted at the beginning of each OFDM symbol to add the guard time. Figure 12 shows the implementation. Here, g extra symbols are added right before the parallel-to-serial converter, so that the parallel-to-serial converter produces a total of (K + g) symbols for each OFDM symbol.

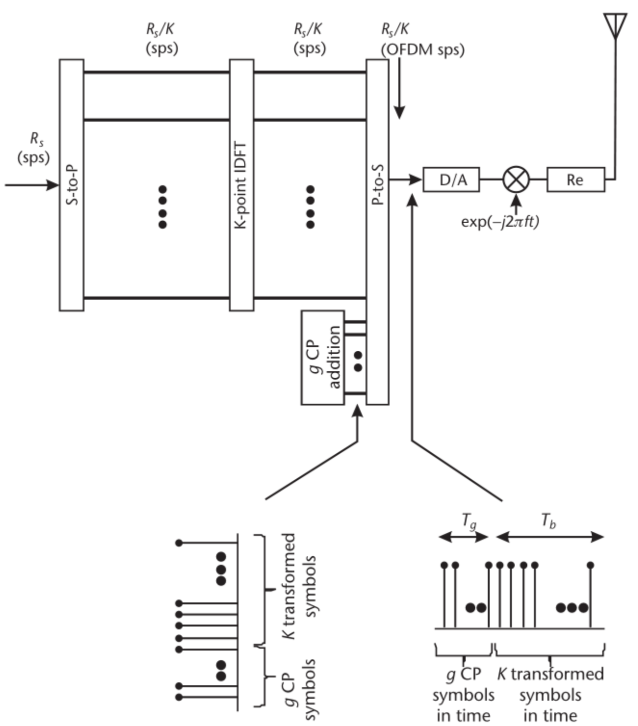


Fig. 12. An OFDM transmitter. CP designates the cycle prefix.

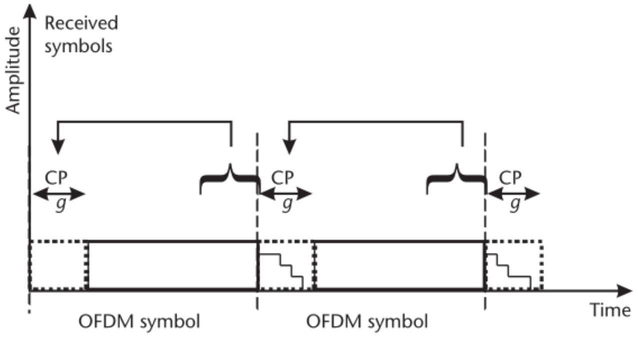


Fig. 13. OFDM symbols with guard time. CP designates the cycle prefix.

Figure(13) shows that when adjacent OFDM symbols are separated by g symbols, the interference between OFDM symbols can be avoided. The guard symbols are also called the cyclic prefix. In practice, the cyclic prefix is generated by simply copying the last g transformed symbols in an OFDM symbol and repeating them at the front of the OFDM symbol. For example, if K = 8, g = 2, and the eight transformed symbols at the output of an 8-point IFFT are {A B C D E F G H}, then the cyclic prefix is G H and the OFDM symbol with the cyclic prefix appended is {G H A B C D E F G H}. If the guard time afforded by the cyclic prefix is larger than the delay spread, then interference between adjacent OFDM symbols can be eliminated.

Another advantage of adding a cyclic prefix is that it turns the channel operation from a linear convolution to a circular convolution, which can be easily implemented using DFT.

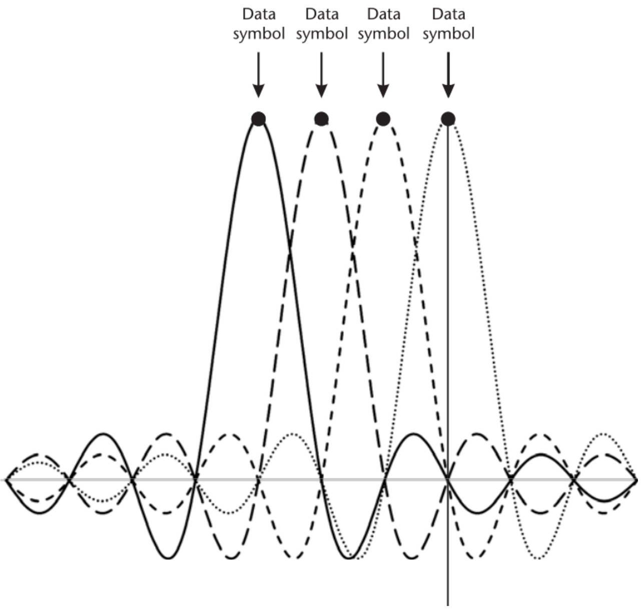


Fig. 14. A set of OFDM subcarriers. The spectrum is shown over the duration of one OFDM symbol

C. Frequency Domain Synchronization

In OFDM, subcarriers overlap in the frequency domain, but data symbols can still be recovered at the receiver because they are sampled at the peaks of the sync functions (see Figure 14).

However, the disadvantage of this arrangement of subcarriers is that it is very sensitive to frequency offset. As can be seen in Figure 14, the peak of a subcarrier has to occur precisely at the zero-crossings of other subcarriers. Any offset would introduce interference from one subcarrier to where the peak of another subcarrier is and to where the data symbol is recovered.

- One cause of frequency offset between the transmitter and the receiver is relative motion between them. Such a motion introduces a **Doppler shift**.
- Another cause of frequency offset is the mismatch of the transmitter and the receiver circuits. Some frequency offset will always be present.

IEEE 802.16e attains frequency synchronization by using symbols that are known a priori. For example, on the downlink, the preamble containing known symbols is used to obtain frequency and timing synchronization; on the uplink, the ranging subchannels transmitting known symbols are used to obtain synchronization.

In addition to the preamble (downlink) and ranging (uplink), the OFDM system itself can be configured to minimize the effect of frequency offset. For a given frequency offset (in hertz), a wider subcarrier bandwidth would help lessen the effect of frequency offset. This is because, given a fixed frequency offset (in hertz), the percent frequency offset (in %) decreases if a subcarrier becomes wider. One way to increase the subcarrier bandwidth is to decrease the number of subcarriers in a given band. However, this option may not be available if a network access provider has already chosen a technology to implement.

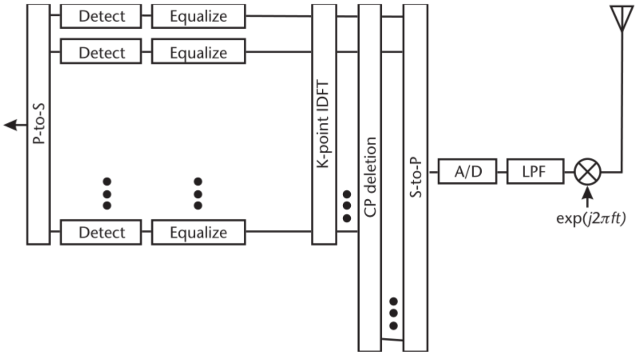


Fig. 15. An OFDM receiver.

D. An OFDM Receiver

A simple OFDM receiver is shown in Figure(15). After downconversion and a low-pass filter (LPF), the signal is now at the baseband but is still continuous in time. The analog-to-digital converter converts baseband continuous-time signals into baseband discrete-time symbols.

The serial-to-parallel converter assembles the incoming symbols into groups of OFDM symbols, each OFDM symbol consisting of K symbols and g cyclic prefix symbols. After throwing away the g cyclic prefix symbols, the remaining K symbols go into the DFT function, which transforms the K symbols in the time domain to K received data symbols in the frequency domain. The equalizer in each path corrects the data symbol carried by the corresponding subcarrier and removes the effects of the channel, and the detector in each path decides what data symbol was actually carried by the corresponding subcarrier. Afterwards, the parallel-to-serial converter rearranges the K parallel substreams of recovered data symbols into a single, high-rate stream of data symbols.

E. Equalization

We know that in the analog world (continuous in time and continuous in frequency) a linear communication system can be modeled by the diagram shown in Figure 16.

In the time domain, the system includes: input signal x(t), impulse response of the channel h(t), and output signal y(t). It is well known that the output signal y(t) is the convolution of the input signal x(t) with the impulse response of the channel h(t), that is,

$$y(t) = x(t) * h(t) \tag{1}$$

Given that convolution in time is multiplication in frequency, we have in the frequency domain:

$$Y(f) = X(f)H(f) \tag{2}$$

where X(f) and Y(f) are the Fourier transforms of x(t) and y(t), respectively; H(f) is the Fourier transform of h(t) and is also known as the transfer function.

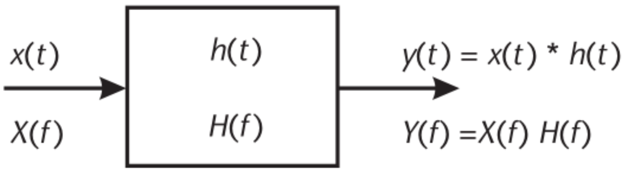


Fig. 16. Channel operation turning input signal x(t) into output signal y(t).

Thus, at the receiver, a simple linear equalization used to recover the input (transmitted) signal in the frequency domain is dividing the output (received) signal by the transfer function, that is,

$$X(f) = \frac{Y(f)}{H(f)} \tag{3}$$

In the digital domain (discrete in time and discrete in frequency), the applicable Fourier transform to use is the discrete Fourier transform (DFT). Specifically, the DFT converting from the discrete-time, time-domain signal x_n to the discrete, frequency-domain signal X_k is:

$$DFT\{x_n\} = X_k = \sum_{n=0}^{N-1} x_n (e^{\frac{-j2\pi kn}{K}}) \tag{4}$$

The IDFT that converts from the discrete, frequency-domain signal X k to the discrete-time, time-domain signal x_n is:

$$IDFT\{X_k\} = x_n = \frac{1}{K} \sum_{k=0}^{K-1} X_k (e^{\frac{j2\pi kn}{K}}) \tag{5}$$

One advantage of using the DFT and the IDFT is that they can be efficiently calculated. In fact, the fast Fourier transform (FFT) and the inverse fast Fourier transform (IFFT) are efficient implementations of the DFT and the IDFT and have enabled many new applications in digital signal processing. Another advantage of the DFT is that it is the only class of Fourier transform that can be finitely parameterized.

Similarly, the convolution-multiplication property of the DFT states that if

$$y_n = x_n \circledast h_n \tag{6}$$

then

$$Y_f = X_f H_f \tag{7}$$

where $\circledast$ denotes circulation convolution. In other words, circular convolution of two signals in time is equivalent to multiplication of DFTs of two signals in frequency. y_n is the circular convolution of x_n and h_n and is operationally

$$y_n = \sum_{m=0}^{K-1} x_{n-m(modK)} h_m \tag{8}$$

Circular convolution is used because the convolution-multiplication property of the DFT requires that x_n is periodic with the period K.In practice, x_n is made to look like a periodic sequence by adding the cyclic prefix. Recall that the cyclic prefix is generated by copying the last g transformed symbols in an OFDM symbol and repeating them at the front of the OFDM symbol. Doing so makes the (K + g) data symbols look periodic, at least for the duration over which circular convolution is performed.

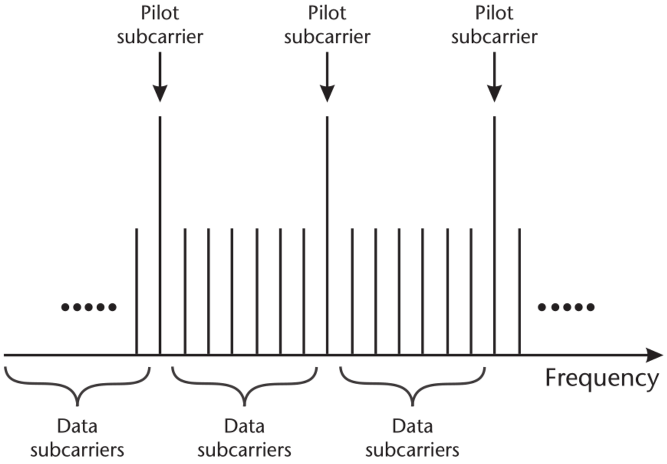


Fig. 17. An example of arrangement of data and pilot subcarriers.

In DFT, being able to perform circular convolution is what makes the relation- ship $Y_k = X_k \ H_k$ true. Once this relationship is true (by adding the cyclic prefix), the effect of the channel is simply to multiply each original data symbol X_k by a complex number H_k, where H_k is the channel response at subcarrier k. Therefore, at the OFDM receiver, a simple linear equalization can be used to recover the input (transmitted) X_k in the frequency domain by just dividing the output (received) Y_k by the channel response H_k , that is,

$$X_k = \frac{Y_k}{H_k} \tag{9}$$

In actuality, the channel also introduces noise n_k ; thus, Eq.9, is rewritten as

$$Y_k = X_k H_k + n_k \tag{10}$$

and the equalized data symbol is:

$$\frac{Y_k}{H_k} = \frac{X_k H_k}{H_k} + \frac{n_k}{H_k} = X_k + \frac{n_k}{H_k} \tag{11}$$

In general, H_k(i.e., the channel) has to be known or at least estimated before the transmitted signal X_k can be recovered. In OFDM, each subcarrier k experiences its own channel response H_k , and the channel response may be different at different frequencies. This means that each subcarrier k requires its own estimated channel response H_k . That is why the OFDM receiver diagram shown previously has K separate equalizers, one for each subcarrier.

In OFDM, a number of the subcarriers are used as pilot subcarriers. Pilot sub-carriers carry known signals, and the receiver can estimate the response of the channel based on what are actually received on the pilot subcarriers. Figure 17 shows an arrangement of pilot subcarriers and data subcarriers.

Figure 17, as an example, shows that there are six data subcarriers between two pilot subcarriers. Typically, the system transmits pilot subcarriers at a higher power to ensure that channel estimates are reliable. Because the channel response may be different at different frequencies, the actual response for a data subcarrier has to be interpolated based on measurements of the two nearest pilot subcarriers. In general, the more pilot subcarriers are provisioned, the more accurate the channel estimates are. However, the obvious tradeoff is that the more pilot subcarriers are provisioned, the fewer subcarriers are available to carry data.

F. OFDM Symbol

Thus far, we have stated that the OFDM spectrum (over the duration of an OFDM symbol) consists of a group of overlapping subcarriers. In exploring the OFDM signal, we will work backwards through the transmitter chain and see how this spectrum of overlapping subcarriers is produced. A multicarrier signal can be produced conventionally by a series of complex multipliers shown in Figure 18.

This series of complex multipliers generates a multicarrier signal x(t) that has the spectrum shown in Figure 19. The delta functions in frequency correspond to the complex sinusoids in time.

The ensemble of complex sinusoids shown in the figure can be characterized by a series of complex data symbols X_k carried by a series of complex subcarriers $\exp(j2\pi fkt)$. In the complex baseband equivalent form, it is

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{j2\pi f_k t} \qquad (12)$$

where f_k is the center frequency of the kth subcarrier and K is the number of subcarriers.

The spectrum of the multicarrier signal consists of delta functions scaled by complex data symbols at f_k .The spectral lines constitute the multicarrier signal because the complex sinusoids $\exp(j2\pi f_k t)$ exist for all time.

Now, let us truncate the multicarrier signal in time so that it exists only for a limited duration T_os , that is,

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{j2\pi f_k t} \quad 0 < T < T_{os} \qquad (13)$$

T_{os} is really the duration of an OFDM symbol.

Limiting any signal to a range in time is equivalent to multiplying it by a rectangular function in time, and multiplication by a rectangular function in time is equivalent to convolution with a sinc function in frequency. If the rectangular function in time lasts T_{os} seconds, then its corresponding sinc function in frequency is 2/T_{os} wide (between the first two zeros).

Therefore, truncating the multicarrier signal in time results in a magnitude spectrum that is the convolution of a series of delta functions with a sinc function. Figure 20 shows that convolution of a series of delta functions with a sinc function results in copies of the sinc function duplicated at where the delta functions used to be.

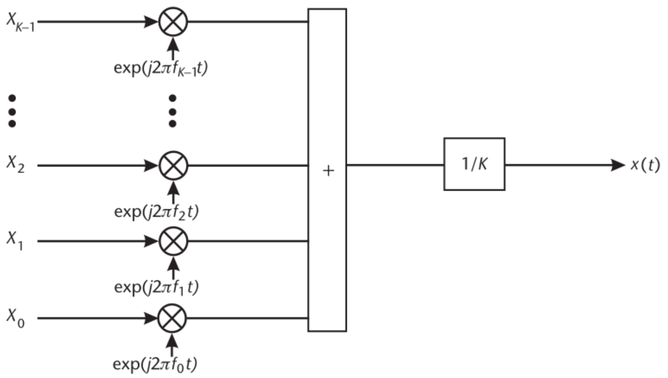


Fig. 18. The magnitude spectrum of the multicarrier signal over the duration of T_{os}.

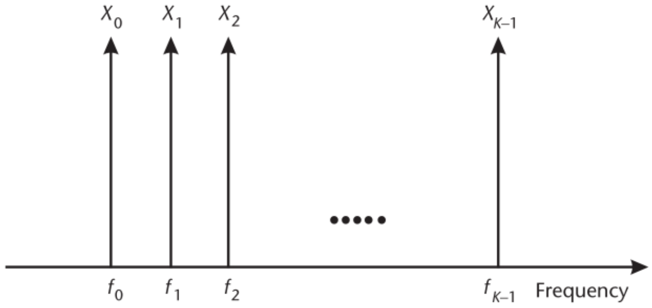


Fig. 19. The magnitude spectrum of the multicarrier signal over the duration of T_{os}.

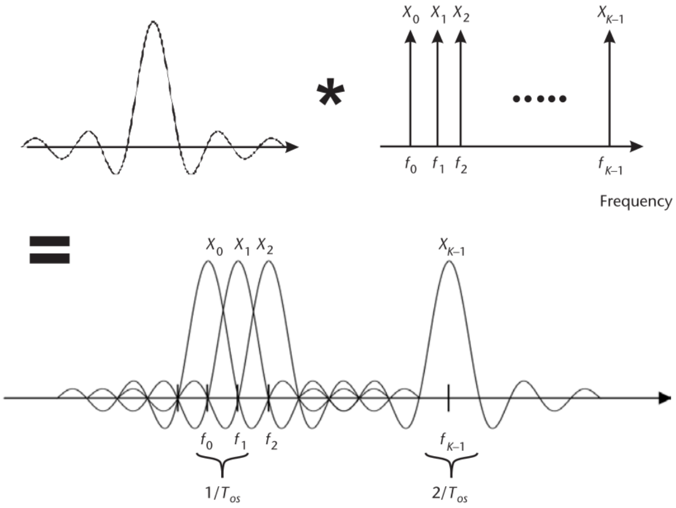


Fig. 20. The magnitude spectrum of the multicarrier signal over the duration of T_{os}.

If the sinc functions overlap, then according to Figure 19 the difference between the centers of two adjacent sinc functions is $1/Tos$, so f_k = k/T_{os} . Substituting k/T_{os} for f_k in the Eq.13, yields:

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kt}{T_{os}}} \quad 0 < T < T_{os} \tag{14}$$

This equation is the continuous-time (analog) version of the multicarrier signal. In other words, it is the signal found at position "B" immediately after the digital-to-analog converter in Figure 10.

To derive the discrete-time (digital) form of the multicarrier signal, one proceeds to sample x(t) in time. Remember that x(t) exists only between t = 0 and t = T_{os} . In the duration of T_{os} seconds, K equally spaced samples are taken in time, so the nth sample takes place at t = $(T_{os}/K)n$. Replacing t with $(T_{os}/K)n$ produces:

$$x(\frac{T_{os}n}{K}) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kT_{os}n}{T_{os}K}} = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \tag{15}$$

$$0 < T < T_{os}$$

which can be written as

$$x(n) = x_n = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \quad 0 < T < T_{os} \tag{16}$$

because the argument of x(·) in discrete time is the sample number n itself.

Due to sampling, Eq.16, is the discrete-time (digital) form of the multicarrier signal. In other words, it is the signal found at position "A" immediately before the digital-to-analog converter in Figure 10. More importantly, one can easily recognize now that Eq.16 for x_n is simply the IDFT of X_k shown previously in Eq.5. What this means is that x_n in time (within an OFDM symbol) can be easily generated by a K-point IDFT function, that is,

$$IDFT\{X_k\} = x_n = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \quad 0 < T < T_{os} \tag{17}$$

Eq.15 is the discrete-time (digital) form of the OFDM signal, over the duration T_{os} , produced by the K-point IDFT. Equation 14 is the continuous-time (analog) version of the OFDM signal, also over the duration T_{os} , after the digital-to-analog conversion. T_{os} , of course, is also known as the duration of the OFDM symbol.

There are three important points to remember regarding the generation of the OFDM signal:

- In the magnitude spectrum of the OFDM signal, the sync functions are present because the multicarrier signal is truncated in time.

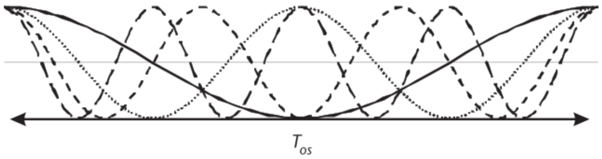


Fig. 21. An illustration of an OFDM symbol in the time domain over the duration of T_os. The OFDM symbol consists of four data symbols.

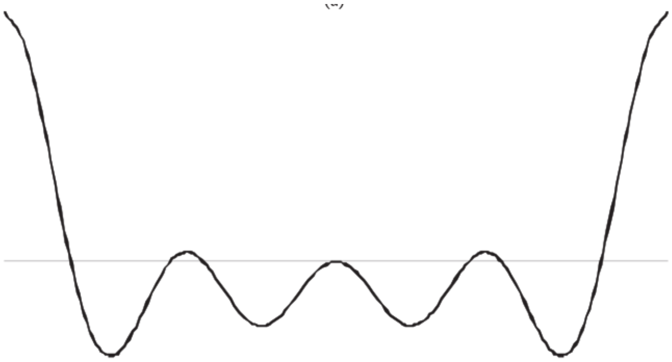


Fig. 22. The actual superposition of four data symbols in time.

- In the magnitude spectrum of the OFDM signal, adjacent sync functions overlap and are separated by $1/T_{os}$ peak-to-peak because the multicarrier signal is truncated in time and limited to a duration of T_{os}.
- Having adjacent sync functions separate by $1/T_{os}$ peak-to-peak also enables the discrete-time version of the OFDM signal to match the IDFT of X_k. This match allows the generation of the OFDM signal using IDFT rather than using many complex multipliers.

Figure 21 illustrates an OFDM symbol in the time domain. The OFDM symbol lasts from t = 0 to t = T_{os} . In particular, Figure 21 shows that this OFDM symbol is made up of four data symbols (four subcarriers), and each subcarrier at a specific frequency is represented by a (truncated) cosinusoid with that frequency. The four subcarriers all have the same magnitude (e.g., 1, 1, 1, 1); thus, the four subcarriers all carry identical data symbols (e.g., 1, 1, 1, 1).

Two important observations can be made regarding this figure. First, in an OFDM symbol, each data symbol lasts the entire T os . Second, these four subcarriers have frequencies $1/T_{os}$,$2/T_{os}$,$3/T_{os}$,$4/T_{os}$ (at baseband); thus:

- The subcarrier with freq $1/T_{os}$ completes one cycle T_{os}
- The subcarrier with freq $2/T_{os}$ completes two cycle T_{os}
- The subcarrier with freq $3/T_{os}$ completes three cycle T_{os}
- The subcarrier with freq $4/T_{os}$ completes four cycle T_{os}

In other words, a subcarrier always completes an integer number of cycles from t = 0 to t = T_{os}.Figure 22 depicts the actual superposition of four data symbols in time over the duration of T_{os} .

It is easy to recognize orthogonality among subcarriers in the frequency domain. In Figure 20, one can see that a data symbol X_k is recovered at the peak of the sync function, and the sync functions are arranged in frequency so that the peak of one sync function is at the zeros of all other sync functions. Because a data symbol X_k is recovered at the peak of the sync function, other sync functions do not interfere with X_k.

While it is straightforward to see orthogonality among subcarriers in the frequency domain, can one quantitatively show that the subcarriers (sync functions shifted by $1/T_{os}$) are orthogonal to each other and do not interfere with each other while in their analog form? To put it another way, can one be sure that the data symbols X_k carried by the subcarriers do not interfere with each other? The answer is yes, and such a proof can be more clearly shown in the time domain. To demonstrate the orthogonality among subcarriers in the time domain, we multiply the analog signal x(t) by the complex conjugate of another subcarrier and integrate over the duration of an OFDM symbol (0 < t < T_{os}):

$$\int_0^{T_{os}} x(t)e^{\frac{-j2\pi lt}{T_{os}}}\,dt \quad 0 < T < T_{os} \tag{18}$$

Note that the complex conjugate of this other subcarrier has an arbitrary frequency l/T_{os} . This integral is evaluated as follows:

$$\begin{aligned}\int_0^{T_{os}} x(t)e^{\frac{-j2\pi lt}{T_{os}}}\,dt &= \frac{1}{K}\int_0^{T_{os}} e^{\frac{-j2\pi lt}{T_{os}}} \sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kt}{T_{os}}}\,dt \\ &= \frac{1}{K}\sum_{k=0}^{K-1} X_k \int_0^{T_{os}} e^{\frac{-j2\pi lt}{T_{os}}} e^{\frac{j2\pi kt}{T_{os}}}\,dt \\ &= \frac{1}{K}\sum_{k=0}^{K-1} X_k \int_0^{T_{os}} e^{\frac{j2\pi(k-l)t}{T_{os}}}\,dt \\ &= \frac{T_{os}X_k}{K} \quad if\, l = k \\ &= 0 \qquad if\, l \neq q \end{aligned} \tag{19}$$

Thus, we see that:

- If the complex conjugate of a subcarrier has the same center frequency l/Tos as the center frequency k/T_{os} of a subcarrier carrying X_k , then the data symbol X_k is recovered.
- Data symbols carried by other subcarriers l ($\neq k$) do not interfere with X_k. In other words, subcarrier k is orthogonal to any other subcarrier l ($\neq q$).

The reason the above expression is 0 if $l \neq k$ is that a subcarrier (in an OFDM signal) always completes an integer number of cycles from t = 0 to t = T_{os} as shown in Figure 21, and the integration of a sinusoid over an integer number of cycles is always 0.

In OFDM systems, the data symbols to be sent are X_k . Recall that OFDM pretends that the data symbols originally exist in the frequency domain. In the receiver, after the receiver receives x_n (plus noise and distortion) in time, it passes x_n (within an OFDM symbol) through the K-point DFT function to recover the original data symbols X_k (plus noise and distortion), that is,

$$DFT\{x_n\} = X_k = \sum_{n=0}^{K-1} x_n(e^{\frac{-j2\pi kn}{K}}) \quad o < T < T_{os} \tag{20}$$

G. Actual OFDM Transmitter

Here we see an actual OFDMA transmitter (and receiver) specified in the IEEE 802.16e standard as a case.Figure 23 shows the basic structure of the transmitter. Note that it is similar to the transmitter shown in Figure 10 with some additions; the additions are shown boldfaced in Figure 23. The stream of information bits from the MAC layer are first fed into the data randomizer. The data randomizer XORs the data bits with bits produced by a shift register. The randomizer has three purposes:

- It scrambles the bits so that a casual eavesdropping receiver cannot easily intercept the data bits.
- It redistributes the bits to avoid long runs of 1s or 0s. A long run of 1s or 0s can cause a subcarrier to become unmodulated.
- It redistributes the bits to avoid long runs of 1s or 0s. A long run of 1s or 0s can cause the received bit stream (at the receiver) to lose synchronization. Bit-level synchronization requires a sufficient number of bit transitions (1-to-0 and 0-to-1) in a given time.

The randomizer only operates on the information bits and is present in both the uplink and the downlink.

After data randomization, the scrambled bits go into the forward error correction (FEC) function, which uses an error-correcting code to add redundancy bits for error correction. In addition to convolutional codes , the IEEE 802.16e standard specifies three additional FEC codes: block turbo code, convolutional turbo code, and low density parity check code. The convolutional code is mandatory in IEEE 802.16e.

After FEC, the coded bits are operated by the interleaver. The purpose of the interleaver is to ensure that the coded bits become sufficiently separated in frequency space and constellation space. In fact, the interleaver operates in two phases. The first phase is for frequency space. In the first phase, consecutive, coded bits are reordered to make sure that these bits are later mapped (by the subcarrier mapper) to nonadjacent subcarriers for frequency diversity. The second phase is for constellation space. In the second phase, consecutive, coded bits are reordered to make sure that these bits are later mapped fairly (by the symbol mapper) to more and less significant bits of the constellation.

After interleaving, the bits are organized into slots. At this point, the system can use the repetition function to further increase the reliability of the transmitted bits. The bits may be repeated by a repetition rate of 2, 4, or 6. Repetition provides a quick way for system designers to trade capacity for coverage. If a system designer would like to increase coverage, he or she can do so by increasing the repetition rate by a desired amount.

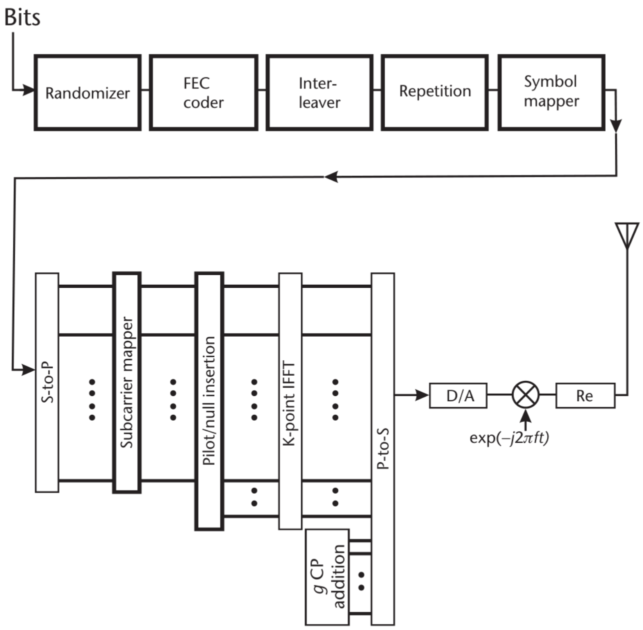


Fig. 23. Basic structure of the OFDMA transmitter in IEEE 802.16e

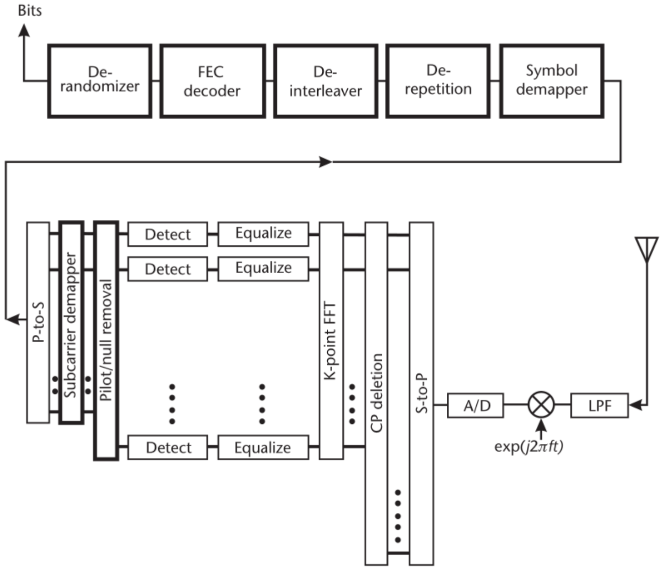


Fig. 24. Basic structure of the OFDMA receiver complementing the transmitter in IEEE 802.16e.

This is because repetition decreases the required SNR at the receiver. However, the tradeoff is reduced capacity because available slots are occupied by repetitive bits .

The symbol mapper maps the (interleaved and/or repetitive) bits to data symbols based on the constellation used at the time (i.e., QPSK, 16-QAM, or 64-QAM). Note that each data symbol (later carried by a corresponding subcarrier) can be a QPSK, 16-QAM, or 64-QAM data symbol depending on the channel condition experienced by that subcarrier. If that subcarrier is experiencing high SINR, then it may carry a 64QAM data symbol to maximize bit rate. If that subcarrier is experiencing low SINR, then it may just carry a QPSK data symbol to ensure reliability.

The serial-to-parallel converter converts the serial stream of data symbols into parallel streams. Then the data symbols go into the subcarrier mapper, which assigns the individual data symbols to the individual subcarriers (i.e., assigning a subcarrier index to each data symbol). The subcarrier mapper is necessary in OFDMA because different data symbols may have come from different users, and assigning data symbols to different subcarriers allows multiple users to access the air interface simultaneously.

The pilot/null insertion function inserts pilot symbols and null symbols. The pilot subcarriers are for channel estimation, and the null subcarriers include the guard subcarriers and the DC subcarrier. The null guard subcarriers and the null DC subcarrier have no power. This way, the (zero-power) guard subcarriers help contain the signal spectrum at the band edges, and the (zero-power) DC subcarrier introduces no DC component to the OFDM signal.

The input to the K-point IFFT consists a total of K parallel symbols, including data symbols, pilot symbols, and null symbols. The K-point IFFT transforms data, pilot, and null symbols from the frequency domain to the time domain. The K transformed symbols, along with g cyclic prefix symbols, go into the parallel- to-serial converter, which produces a serial output of transformed symbols in the time domain. At the output of the parallel-to-serial converter, the block of K transformed symbols constitutes an OFDM symbol, and the g CP symbols constitute the cyclic prefix.

The digital-to-analog converter changes the discrete-time symbols to analog signals, which are upconverted and transmitted over the air.

H. Actual OFDM Receiver

Figure 24 shows the basic structure of the receiver. At the receiver, the reverse of the process of Figure 23 takes place. The 802.16e standard specifies, in detail, what the transmitter does and contains. However, as has become the convention, the standards do not explicitly specify the architecture of the receiver. This is done to leave some details of the end-to-end implementation to the vendor, and many vendors differentiate themselves by how they implement their receivers. Figure 23 depicts one possible implementation of an OFDMA receiver.

After downconversion and analog-to-digital conversion, the received discrete-time symbols at baseband go into the serial-to-parallel converter, which converts the serial stream of received symbols into parallel streams. The g cyclic prefix symbols are removed, and the remaining K symbols go into the K-point FFT function. The K-point FFT function transforms the K symbols in the time domain to the K data (and pilot and null) symbols in the frequency domain.

The equalizer (in each path) takes the effects of the channel out of each received data symbol and corrects the received data symbol. Then the detector (in each path) estimates what the original data symbol is . The pilots are read and used for channel estimation, and then both the pilot symbols and the null symbols are removed. What remain are the recovered data symbols.

The subcarrier demapper rearranges the recovered data symbols in parallel back in the order by which users were originally assigned (to the individual subcarriers). In effect, the subcarrier demapper assigns the individual data symbols (recovered in the frequency domain) back to the individual users.

The parallel-to-serial converter rearranges the parallel substreams of recovered data symbols into a single, high-rate stream of data symbols. This high-rate stream of data symbols then go into the symbol demapper, which matches each data symbol in the stream to the bit pattern that data symbol represents. The resulting high-rate stream of bits then go through derepetition, deinterleaver, FEC decoder, and derandomizer, which constitute the inverse of the first four functions in the transmitter.

I. Advantages of OFDM

Because OFDM also transmits using multiple narrowband subcarriers, it is robust against ISI and multipath fading, can adjust modulation and coding for each subcarrier, and has simple equalizers. In addition, OFDM offers two more important advantages:

- OFDM has low-complexity modulation; and
- OFDM achieves a better spectral efficiency than conventional FDM.

IV. OFDMA

A. Basics of OFDMA

Whereas OFDM assigns one block (in time) to one user, OFDMA is a method that assigns different groups of subcarriers (in frequency) to different users. This way, more than one user can access the air interface at the same time. Recall that in OFDM all K subcarriers are used to carry data for one user only. OFDM assigns all subcarriers to a single user at the same time, and only one user can transmit at a time. If multiple users want to transmit using OFDM, then those users have to take their turns in time. For example, in OFDM each user can be assigned one OFDM symbol in time, and OFDM symbols are assigned to their respective users before OFDM symbols enter the OFDM transmitter.

In OFDMA, instead of sequentially assigning OFDM symbols in time to different users, the system directly assigns subcarriers in frequency to different users. Figure 25 shows a simplified OFDMA transmitter. The high-rate stream of baseband data symbols is still running at a rate of $R_s sps$, and each data symbol lasts $1/R_s$ s. This high-rate stream consists of J groups of complex data symbols; each group contains L complex data symbols, and each group (of L data symbols) is later assigned to a different user. So there are a total of JL complex data symbols in J groups.

The serial-to-parallel converter assigns the high-rate stream into JL separate low-rate substreams; each low-rate substream has a rate of R_s/JL sps. In doing so, the serial-to-parallel converter assigns successive data symbols (at its input) to JL separate low-rate substreams (at its outputs). So at any given time at the output of the serial-to-parallel converter, there is a set of JL data symbols in parallel.

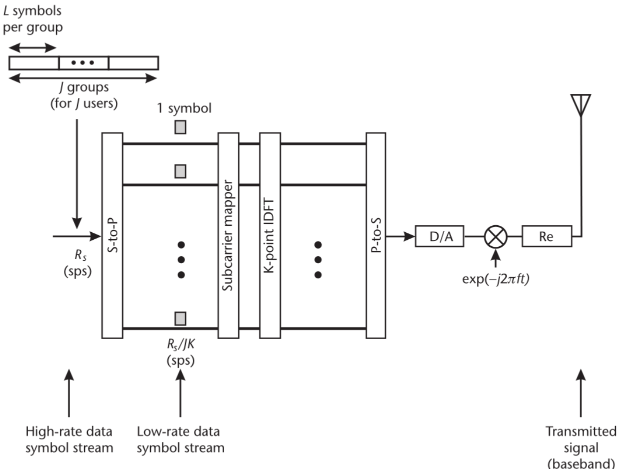


Fig. 25. An OFDMA transmitter

The subcarrier mapper maps JL data symbols to their respective subcarriers (which are assigned to different users). Specifically, the subcarrier mapper assigns J groups of data symbols to J users in frequency. In effect, the subcarrier mapper reorders the parallel data symbols according to the particular subcarriers assigned to each user.

The set of mapped JL(= K) data symbols in parallel pass through the IDFT function, which transforms the K data symbols. The K transformed symbols in K substreams then pass through the parallel-to-serial converter that puts the K transformed symbols in series. This block of K transformed symbols in series constitutes a single OFDM symbol. Successive OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K OFDM symbols per second, and each OFDM symbol lasts $K/R_s s$.

Figure 26 shows the spectrums of the high-rate stream, the low-rate sub-streams, and the transmitted signal. In particular, the spectrum of the transmitted signal is shown over J groups of data symbols, or one OFDM symbol, only. The subcarriers are spaced so that they overlap in frequency but are orthogonal because a data symbol is recovered at the peak of a subcarrier. Note that, in this case, data symbols belonging to a user are carried by contiguous subcarriers.

In general, there are two ways to assign users' data symbols to subcarriers: distributed and contiguous. In a distributed subcarriers arrangement, subcarriers are assigned pseudorandomly to users. In a contiguous subcarriers arrangement,subcarriers are assigned to users in continuous sets (this is the scheme shown in Figure 26).

The subcarrier mapper shown in Figure 23 is key in implementing OFDMA because it is the subcarrier mapper that assigns users to subcarriers. More accurately, it is the subcarrier mapper that assigns users' data symbols to subcarriers.

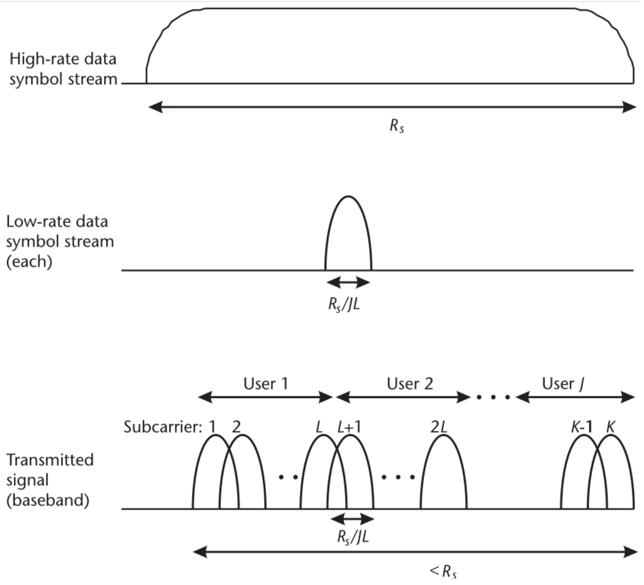
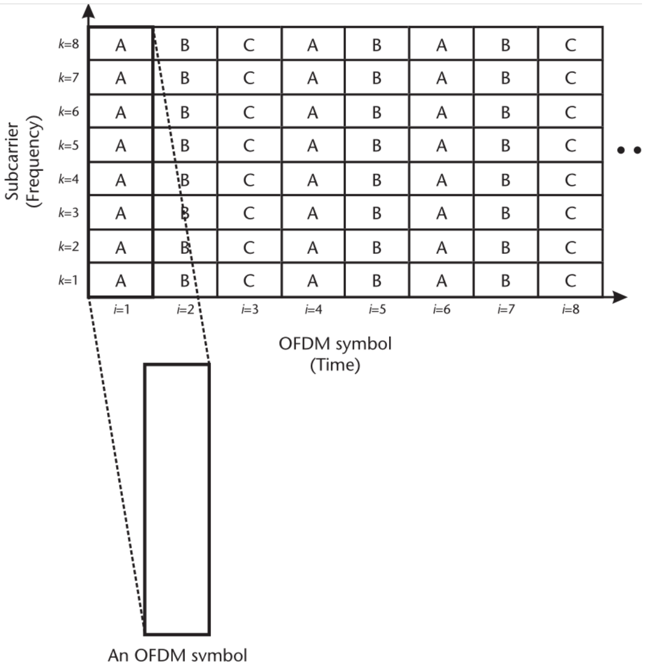


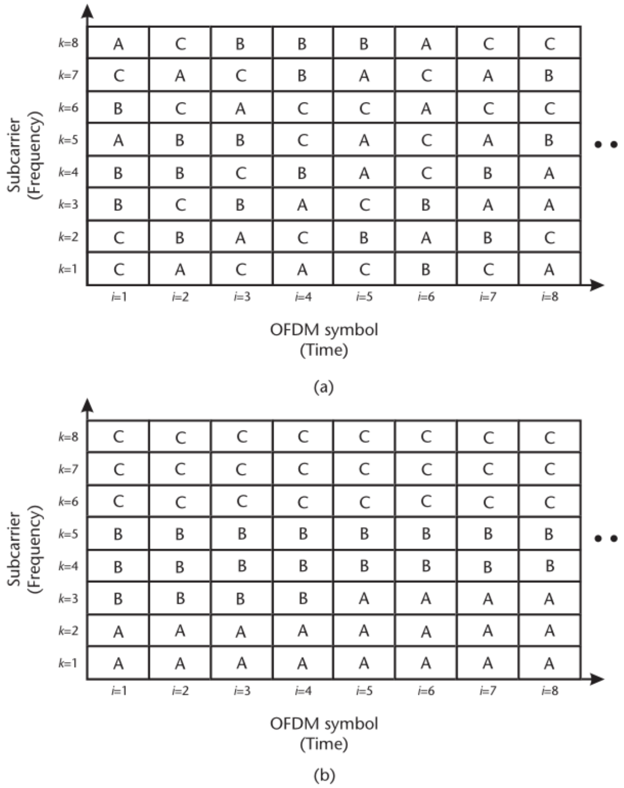
Fig. 26. The spectrums of the high-rate data symbol stream, the low-rate data symbol substreams, and the transmitted OFDMA signal. The spectrum of the transmitted OFDMA signal is shown for the duration of an OFDM symbol (J groups). The figure shows that data symbols belonging to a user are carried by contiguous subcarriers.



	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
k=8	A	B	C	A	B	A	B	C
k=7	A	B	C	A	B	A	B	C
k=6	A	B	C	A	B	A	B	C
k=5	A	B	C	A	B	A	B	C
k=4	A	B	C	A	B	A	B	C
k=3	A	B	C	A	B	A	B	C
k=2	A	B	C	A	B	A	B	C
k=1	A	B	C	A	B	A	B	C

Fig. 27. Multiple users using OFDM.

Figure 27 and Figure 28 show a train of OFDM symbols along both time and frequency dimensions, and these figures compare and contrast OFDM and OFDMA in a situation where three users (A, B, and C) would like to access the air interface. There are a total of eight subcarriers, and the figures show how these users are assigned to subcarriers as a function of time.



(a)

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
k=8	A	C	B	B	B	A	C	C
k=7	C	A	C	B	A	C	A	B
k=6	B	C	A	C	C	A	C	C
k=5	A	B	B	C	A	C	A	B
k=4	B	B	C	B	A	C	B	A
k=3	B	C	B	A	C	B	A	A
k=2	C	B	A	C	B	A	B	C
k=1	C	A	C	A	C	B	C	A

(b)

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
k=8	C	C	C	C	C	C	C	C
k=7	C	C	C	C	C	C	C	C
k=6	C	C	C	C	C	C	C	C
k=5	B	B	B	B	B	B	B	B
k=4	B	B	B	B	B	B	B	B
k=3	B	B	B	B	A	A	A	A
k=2	A	A	A	A	A	A	A	A
k=1	A	A	A	A	A	A	A	A

Fig. 28. Multiple users using OFDMA: (a) distributed subcarriers and (b) contiguous subcarriers.

Figure 28 shows the situation in OFDMA. In OFDMA, the subcarrier mapper assigns different users to different subcarriers at a time. In general, there are two ways of assigning users to subcarriers: distributed subcarriers and contiguous subcarriers. A logical set of subcarriers is sometimes called a subchannel. (Each user may be assigned one or more subchannels.)

Figure 28(a) illustrates the arrangement of distributed subcarriers, where users are assigned pseudorandomly to subcarriers. For example, in the first OFDM symbol, user A is assigned two subcarriers (k = 5, 8), whereas user B is assigned three subcarriers (k = 3, 4, 6). Alternatively, in the first OFDM symbol, user A's two data symbols are carried by subcarrier k = 5 and subcarrier k = 8, whereas user B's three data symbols are carried by subcarrier k = 3, subcarrier k = 4, and subcarrier k = 6. Distributing subcarriers pseudorandomly affords frequency diversity (to a single user)

Figure 28(b) illustrates the arrangement of contiguous subcarriers. In contiguous subcarriers, subcarriers are assigned to users in continuous groups. For example, in the first OFDM symbol, user A is assigned two subcarriers (k = 1, 2),user B is assigned three subcarriers (k = 3, 4, 5), and user C is assigned three subcarriers (k = 6, 7, 8). Contiguous subcarriers can take advantage of multiuser diversity.

IEEE 802.16e has certain ways of arranging users' data symbols in time and frequency, called permutation modes.

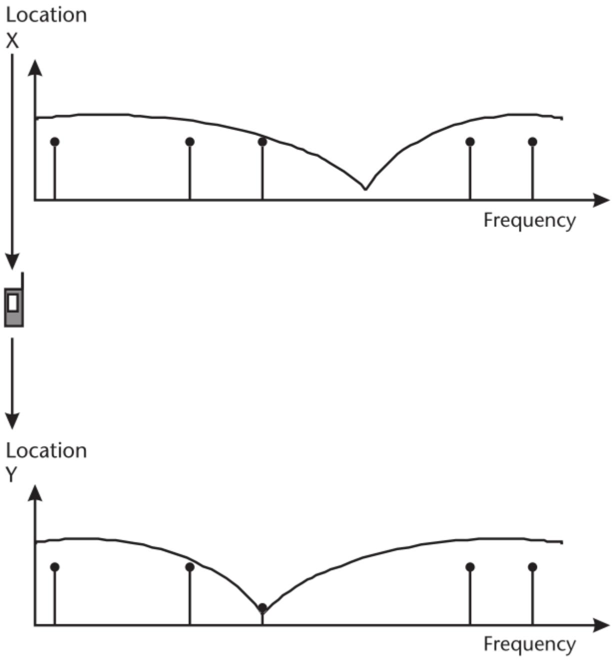


Fig. 29. mobile user travels from location X to location Y. At location X, none of the user's subcarriers is degraded by the channel response. At location Y, a subcarrier is experiencing deep fade.

B. Frequency diversity

Frequency diversity is achieved by forming a subchannel through distributed subcarriers. For distributed subcarriers, the subcarrier mapper pseudorandomly distributes a user's subcarriers across the band. As far as a single user is concerned, such a distribution of subcarriers offers frequency diversity. This is so because if a user's subcarriers are distributed pseudorandomly, some of its subcarriers likely would not experience fades while some of its other subcarriers would.

Frequency diversity afforded by distributed subcarriers is well suited for mobile users because as a mobile user changes its location, the user experiences different multipath fadings at different locations. Figure 29 illustrates. For a user travelling from location X to location Y, that user experiences two different channel responses at the two locations. If a user's subcarriers are distributed across the band, some of its subcarriers may avoid fading.

C. MultiUser Diversity

Multiuser diversity occurs when different users at different locations experience different channel responses. This form of diversity can be achieved by forming a subchannel through contiguous subcarriers. For contiguous subcarriers, a group of adjacent subcarriers are assigned to a single user. This scheme cannot take advantage of frequency diversity because all of the user's subcarriers are in the same vicinity of the spectrum. If a deep fade falls on top of those subcarriers belonging to a user, then that user will experience degraded channel. However, contiguous subcarriers can offer multiuser diversity.

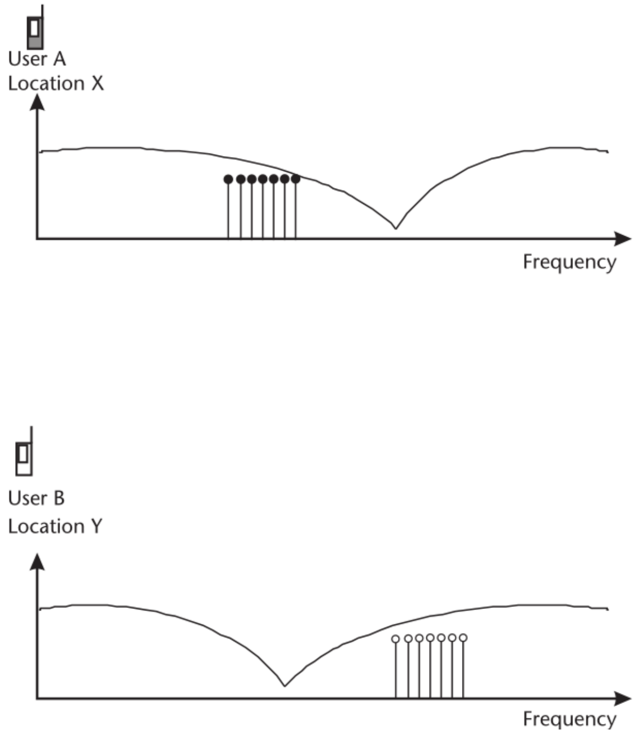


Fig. 30. User A is at location X and user B is at location Y. At location X, user A's subcarriers are experiencing a good channel response (away from the null). At location Y, user B's subcarriers are also experiencing a good channel response (away from the null).

Multiuser diversity afforded by contiguous subcarriers is well suited for fixed users because a fixed user's location does not change, so the channel response experienced by the user is relatively constant. This way, the system can assign a user a set of contiguous subcarriers based on the user's channel response. Figure 30 shows the case for two users. User A is fixed at location X, and user B is fixed at location Y. The two users experience two different channel responses because they are at two different locations. Therefore, the system can assign user A a set of contiguous subcarriers where user A is experiencing a good channel response, and it can assign user B a different set of contiguous subcarriers where user B has a good channel response.

Typically, the base station measures the channel response across the band by using pilot subcarriers that locate throughout the band. In OFDMA systems, it is possible to assign subcarriers based on their SINRs. For contiguous carriers, the base station can assign to a user a set of contiguous subcarriers that experience high SINR.

For a fixed user, contiguous subcarriers are especially helpful on the uplink in conserving the user's battery power. By transmitting on those subcarriers that experience strong SINR, the user device does not have to expend much power to attain a desired uplink bit rate.

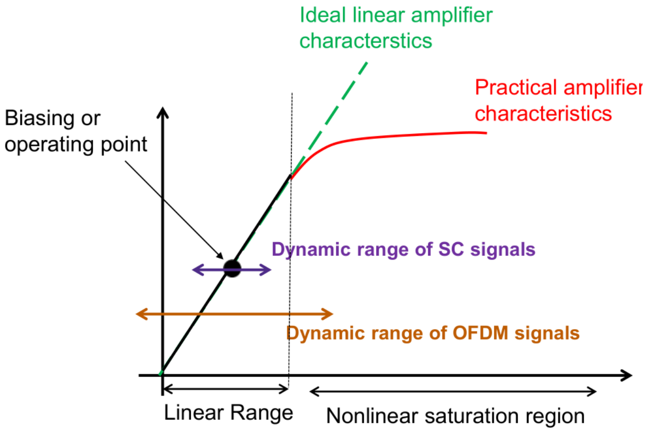


Fig. 31. Non-linear amplifier characteristics

D. Remarks

In dynamically assigning subcarriers to users, the subcarrier mapper performs an important scheduler function, which "schedules" different subcarriers to carry different users' data symbols. For example, in the first four OFDM symbols shown in Figure 28(b), user A is allocated less bandwidth than user B. However, in the second four OFDM symbols shown in the same figure, user A is allocated more bandwidth than user B. This change in granted bandwidth as a function of time is probably due to different users' bandwidth requests at different times subject to any quality-of-service (QoS) constraints. In fact, the scheduler makes optimizations decisions on the assignments of subcarriers and allocation of bandwidth resources based on multiple factors, including a user's current request for bandwidth, other users' pending (and competing) requests for bandwidth, users' QoS requirements, and channel quality experienced by each user.

Because OFDMA can dynamically allocate resources both in frequency (subcarriers) and in time (OFDM symbols), it is expected that broadband mobile systems will mostly use the more flexible OFDMA in the physical layer. In fact, the Mobile WiMAX System Profile specifies only the OFDMA implementation at the physical layer because OFDMA's scalable architecture is more suitable for mobile usage.

V. ISSUES OF OFDM AND OFDMA

- Frequency synchronization
- Dynamic Range of OFDM symbols (see figure 31). Amplifiers with high linear-range are costly.Operating in non-linear region causes distorion.

VI. APPLICATIONS OF OFDMA

- OFDM is used in Wi-Fi,ADSL internet access,4G wireless communications, and digital television and radiobroadcast services.
- Earlier, lower-speed versions of the IEEE 802.11 standards, such as IEEE 802.11b, use DSSS at the physical layer. Later, higher-speed versions, such as IEEE 802.11g and IEEE 802.11n, predominantly use OFDM.

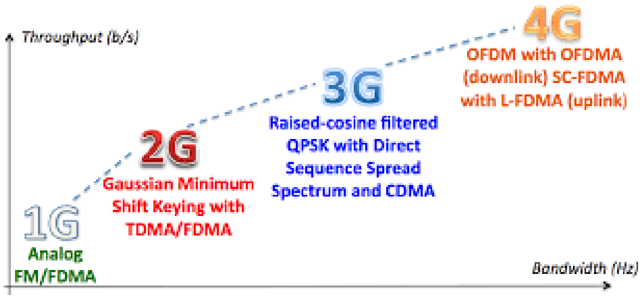


Fig. 32. Evolution of Generations in Mobile commmunications

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