OFDM and OFDMA

Sai Krishna Charan D   Sasi Kiran D

April 2020

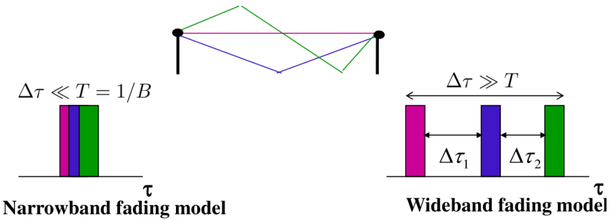
Fading Models

Narrow Band Fading Models

- Paths are non-resolvable
- No problem caused due to multipath because $\Delta\tau << T$.

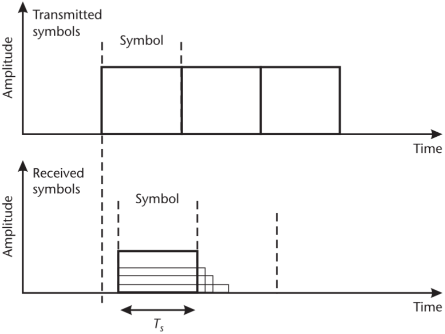
Wide Band Fading Models

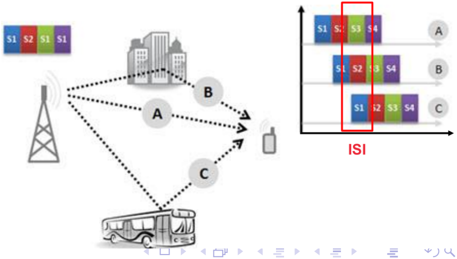
- Multipath components are resolvable
- Time domain - multipath leads to the **spreading out** of the arrival time of received signals



Inter Symbol Interference

- In a high-speed wireless system, R_s is high, symbol time T_s is low.
- As Symbol rate R_s becomes higher and the symbol time T_s becomes shorter, eventually T_s can become much shorter than the channel delay spread τ (i.e., $T_s << \tau$) for a given channel.
- The delayed versions of one symbol start to leak into and interfere with the subsequent symbol.





Motivation for MultiCarrier Subsystems

- Transmitting NarrowBand Subcarriers
- Dividing the high-rate symbol stream into many low-rate symbol substreams, each with a lower symbol rate R_s/K (K is number of subcarriers).
- Symbol time of each low-rate symbol substream becomes T_sK.
- From the perspective of each low-rate substream of data symbols, each symbol experiences little ISI.
- Because when the symbol time of a symbol in the substream becomes large relative to the channel delay spread τ (i.e., $T_sK >> \tau$), delayed versions of a symbol have little effect on the next symbol.

Motivation for MultiCarrier Subsystems

Frequency Selective Fading

- Multipath leads to “nulls” in the frequency response of the channel in frequency.
- Multipath fading is also known as frequency- selective fading.
- A frequency-selective channel is characterized by the coherence bandwidth Wc , which is the bandwidth over which the channel appears relatively flat and unvarying.
- Narrowband subcarriers addresses the problem of frequency-selective fading.
- Each narrowband subcarrier has a much smaller bandwidth (than the original wideband carrier)
- Each narrowband sub-carrier can be said to undergo flat fading.
- Bandwidth R_s/K of a subcarrier becomes small relative to the channel coherence bandwidth W_c (i.e., $R_s/K << W_c$), frequency-selective fading is substantially reduced.

Flat Fading

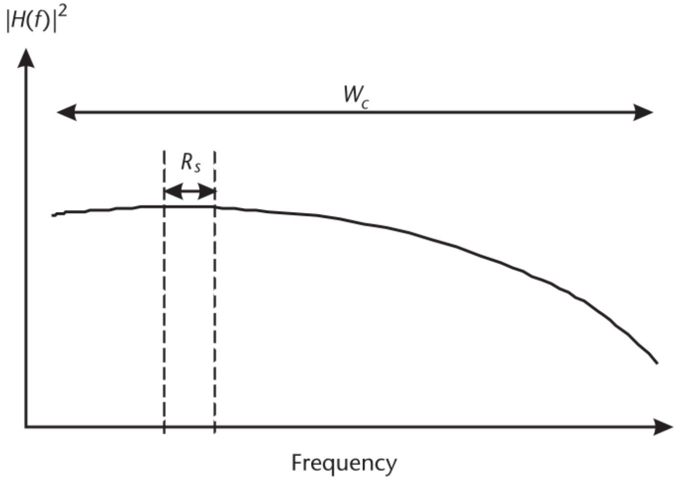


Figure: By keeping the bandwidth of a subcarrier much less than the coherence bandwidth, the subcarrier is experiencing an approximately flat channel.

Other Advantages of MultiCarrier Subsystems

- Modulation techniques and coding per subcarrier can be incorparated for better performance.
- For those subcarriers that experience little fades, they can take advantage of a more efficient modulation scheme (e.g., 16-quadrature amplitude modulation or 16-QAM) and/or higher-rate error correction code (e.g., rate 3/4 convolutional code).
- For those subcarriers that experience fades, they can fall back to a more robust modulation (e.g., quadrature phase shift keying or QPSK) and/or lower-rate error correction code (e.g., rate 1/3 convolutional code).
- By **adapting modulation** and **coding for each subcarrier**, the system can achieve the best possible overall capacity and performance.

Contd...

- .When a subcarrier is narrow, the required equalization function for that subcarrier in the receiver is simpler.
- Note that in a receiver, a channel equalizer is still required for each subcarrier. Thus a total of K (albeit simpler) equalizers are needed in the receiver.
- The robustness against ISI and multipath fading is the key advantage of using multiple narrowband subcarriers, and this advantage is carried into FDM,FDMA,OFDM and OFDMA.

- Frequency-division multiplexing (FDM) is an analog technique that can be applied when the bandwidth of a link) is greater than the combined bandwidths of the signals to be transmitted.

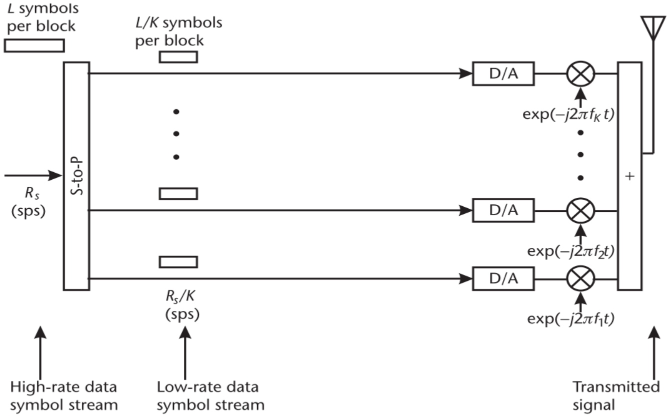


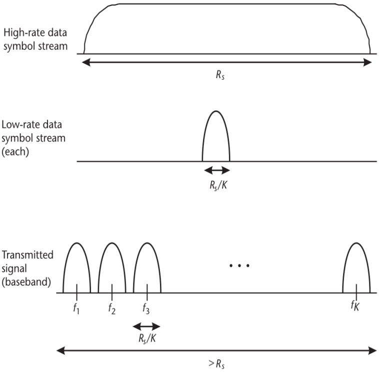
Figure: A conventional FDM transmitter

Conventional FDM Contd..

- At the input of the transmitter, there is a single high-rate stream of baseband data symbol running at a rate of R_s.
- Each block contains L complex data symbols.
- A serial-to-parallel (S-to-P) converter converts the high-rate stream into K separate low-rate substreams.
- Each low-rate substream has a rate of Rs/K sps.
- Serial- to-parallel converter breaks the one large block containing L symbols into K smaller blocks in parallel, each containing L/K data symbols.
- Because of the lower rate, each symbol in the substream lengthens to K/R_s s.
- Each low-rate substream is modulated by a complex sinusoid $\exp(j2\pi f_k t)$ to a different frequency f_k
- After summation, the composite signal consists of K signals multiplexed in the frequen domain.

Conventional FDM Contd..

- As a result, K separate low-rate, narrowband subcarriers are used to transmit the original high-rate, wideband stream.
- To minimize interference between subcarriers, a guard band is placed between two adjacent subcarriers.
- The total bandwidth occupied by the K subcarriers is greater than simply K times the bandwidth of each subcarrier.



Advantages & Disadvantages

Advantages

- It is effective at combating intersymbol interference (ISI) and multipath fading and has simple equalization.
- It can adjust modulation and coding for each subcarrier

Disadvantages

- The transmitter needs to have K separate D-to-A converters and K separate radio frequency (RF) modulators.
- FDM is not bandwidth efficient. The extra guard bands necessarily add to the total bandwidth requirement.

Why OFDM?

So is there a way to address these disadvantages and, at the same time, retain the advantage of transmitting multiple narrowband subcarriers? The answer is yes - in the form of **OFDM**.

OFDM

- OFDM overcomes the problem of the large bandwidth requirement imposed by guard bands.
- Instead of using K local oscillators (LOs) and K multipliers in modulation, OFDM uses a mathematical technique called **discrete Fourier transform (DFT)** to generate the subcarriers.
- The subcarriers generated this way do not need additional guard bands and can be placed closer together in the frequency domain.
- The subcarriers are also orthogonal to each other over a set duration (i.e., over the duration of an OFDM symbol).
- DFT and its inverse can be efficiently computed, eliminating the need for separate RF components for separate subcarriers.

OFDM Transmitter

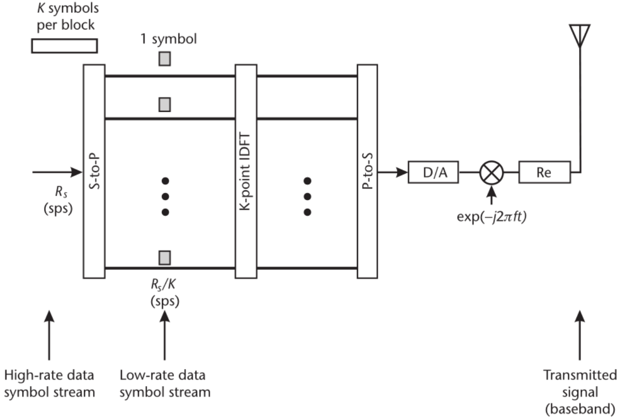


Figure: An OFDM transmitter

OFDM Transmitter Contd..

- The high-rate stream of data symbols is still running at a rate of R_s sps, and each data symbol lasts $1/R_s$ s.
- High-rate stream of data symbols consists of blocks of complex data symbols, and each block contains K complex data symbols.
- Serial-to-parallel converter converts the high-rate stream into K separate low-rate substreams.
- Serial-to-parallel converter assigns successive data symbols (at its input) to K separate substreams (at its outputs).
- Set of K data symbols in parallel pass through the inverse DFT (IDFT) function, which transforms the K data symbols. After IDFT, the K transformed symbols in the K substreams then pass through the parallel-to-serial (P-to-S) converter that puts the K transformed symbols in series.

OFDM Symbol

- This block of K transformed symbols in series constitutes a single block or an **OFDM symbol**
- Successive OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K OFDM symbols per second, and each OFDM symbol lasts $K/R_s s$.

Subcarrier Spacing

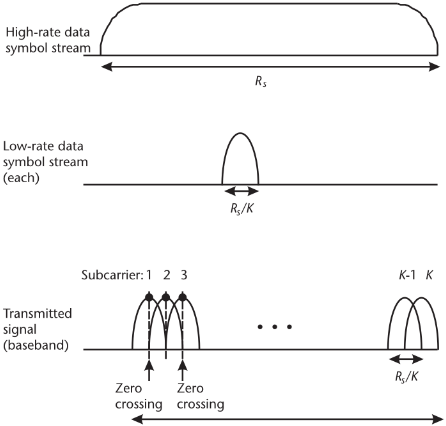


Figure: The spectrums of the high-rate data symbol stream, the low-rate data symbol sub- streams, and the transmitted OFDM signal. The spectrum of the transmitted OFDM signal is shown for the duration of one OFDM symbol. Note that the first zero crossings of subcarrier 2 fall on the peaks of adjacent subcarriers 1 and 3.

Orthogonal?

- Because OFDM recovers the data symbol at the peak of each subcarrier, the subcarriers are orthogonal to each other and there is no interference — hence the term orthogonal FDM (OFDM).
- Although there are K different subcarriers, all K subcarriers in a block (an OFDM symbol) are assigned to only one user. In other words, only one user transmits in a block (an OFDM symbol).

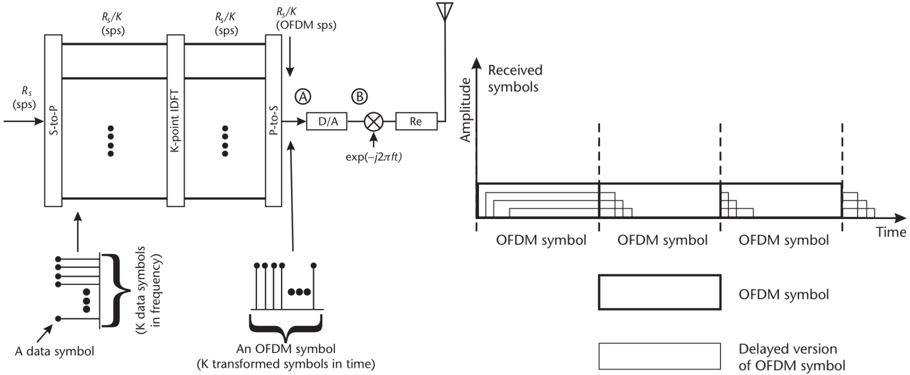
Need Guard time?

- The IDFT function transforms the set of K parallel data symbols from the frequency domain into the time domain. In OFDM, the system pretends that the data symbols originally exist in the frequency domain. That is why later at the receiver,the data symbols are recovered at the peaks of the (overlapping) sinc functions in the frequency domain.
- A set of K transformed symbols in series is called an OFDM symbol and the OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K .

Inter Block Interference

- Because of the multipath, delayed versions of an OFDM symbol can fall on the next OFDM symbol. As a result, there is inter-OFDM symbol interference (i.e., interblock interference, or IBI) between adjacent OFDM symbols.(With no guard time).

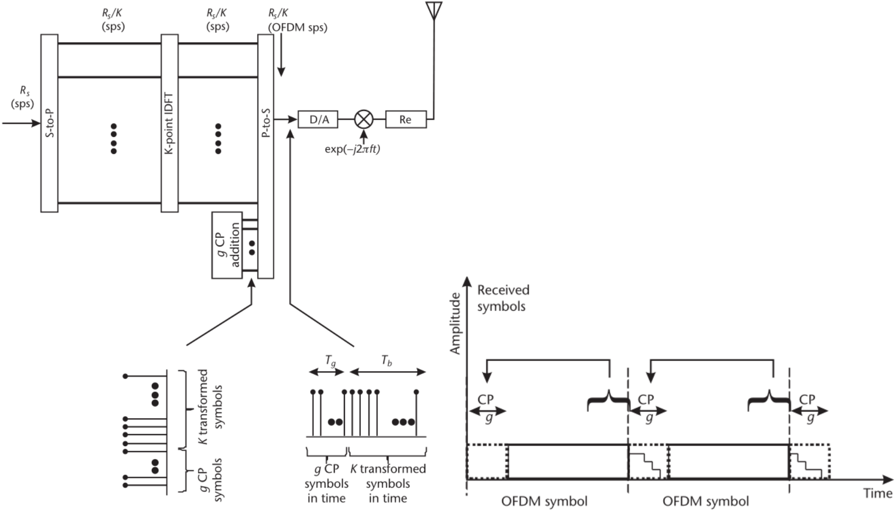
Inter OFDM symbol Interference(IBI)



Using Cyclic Prefeix to avoid IBI

- An advantage of OFDM is that the data symbols in a single OFDM symbol do not interfere with one another inside an OFDM symbol. To state it in another way, the K data symbols do not affect one another within the symbol time of an OFDM symbol as far as data recovery at the receiver is concerned. This is because data are recovered at the peaks of the overlapping sinc functions (in the frequency domain).
- To reduce such interference between adjacent OFDM symbols, one needs to add an extra guard time between adjacent OFDM symbols.
- In practice, extra symbols are inserted at the beginning of each OFDM symbol to add the guard time. g extra symbols are added right before the parallel-to-serial converter, so that the parallel-to-serial converter produces a total of $(K + g)$ symbols for each OFDM symbol.

Insertion of Cyclic Prefix



Example of Cyclic Prefix

- The guard symbols are also called the cyclic prefix. In practice, the cyclic prefix is generated by simply copying the last g transformed symbols in an OFDM symbol and repeating them at the front of the OFDM symbol.
- For example, if K = 8, g = 2, and the eight transformed symbols at the output of an 8-point IFFT are {A B C D E F G H}, then the cyclic prefix is G H and the OFDM symbol with the cyclic prefix appended is {G H A B C D E F G H}.
- If the guard time afforded by the cyclic prefix is larger than the delay spread, then interference between adjacent OFDM symbols can be eliminated.

Linear Convolution => Circular Convolution

- Another advantage of adding a cyclic prefix is that it turns the channel operation from a linear convolution to a circular convolution, which can be easily implemented using DFT.

Frequency Domain Synchronization

- In OFDM, subcarriers overlap in the frequency domain, but data symbols can still be recovered at the receiver because they are sampled at the peaks of the sync functions

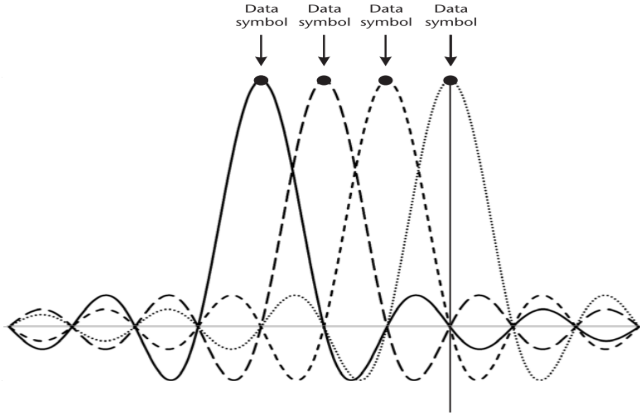


Figure: A set of OFDM subcarriers. The spectrum is shown over the duration of one OFDM symbol

Sensitivity to frequency offset

- Disadvantage of this arrangement of subcarriers is that it is very sensitive to frequency offset.
- The peak of a subcarrier has to occur precisely at the zero-crossings of other subcarriers. Any offset would introduce interference from one subcarrier to where the peak of another subcarrier is and to where the data symbol is recovered.
- One cause of frequency offset between the transmitter and the receiver is relative motion between them. Such a motion introduces a **Doppler shift**.
- Another cause of frequency offset is the mismatch of the transmitter and the receiver circuits. Some frequency offset will always be present.

Need for frequency synchronization!

- IEEE 802.16e attains frequency synchronization by using symbols that are known a priori. For example, on the downlink, the preamble containing known symbols is used to obtain frequency and timing synchronization; on the uplink, the ranging subchannels transmitting known symbols are used to obtain synchronization.
- For a given frequency offset (in hertz), a wider subcarrier bandwidth would help lessen the effect of frequency offset. This is because, given a fixed frequency offset (in hertz), the percent frequency offset (in %) decreases if a subcarrier becomes wider.
- One way to increase the subcarrier bandwidth is to decrease the number of subcarriers in a given band.

OFDM Recevier

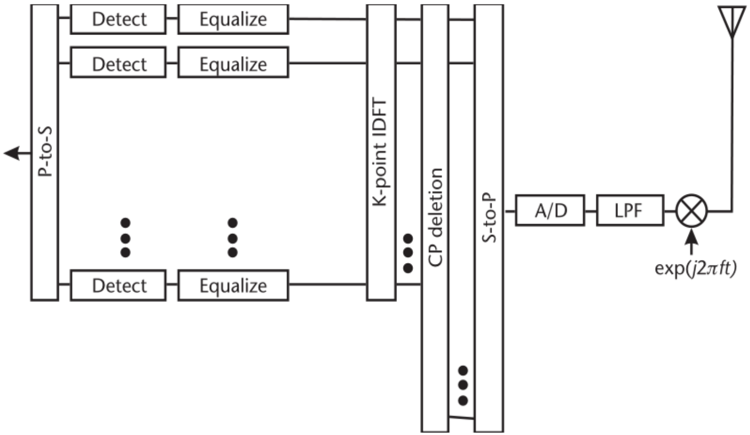


Figure: An OFDM receiver.

OFDM Receiver Functions

- After downconversion and a low-pass filter (LPF), the signal is now at the baseband but is still continuous in time. The analog-to-digital converter converts baseband continuous-time signals into baseband discrete-time symbols.
- The serial-to-parallel converter assembles the incoming symbols into groups of OFDM symbols, each OFDM symbol consisting of K symbols and g cyclic prefix symbols.
- After throwing away the g cyclic prefix symbols, the remaining K symbols go into the DFT function, which transforms the K symbols in the time domain to K received data symbols in the frequency domain.
- The equalizer in each path corrects the data symbol carried by the corresponding subcarrier and removes the effects of the channel, and the detector in each path decides what data symbol was actually carried by the corresponding subcarrier.
- Afterwards, the parallel-to-serial converter rearranges the K parallel substreams of recovered data symbols into a single, high-rate stream of data symbols.

Equalization

- In the digital domain (discrete in time and discrete in frequency), the applicable Fourier transform to use is the discrete Fourier transform (DFT). Specifically, the DFT converting from the discrete-time, time-domain signal x_n to the discrete, frequency-domain signal X_k is:

$$DFT\{x_n\} = X_k = \sum_{n=0}^{N-1} x_n (e^{\frac{-j2\pi kn}{K}}) \tag{1}$$

- The IDFT that converts from the discrete, frequency-domain signal X k to the discrete-time, time-domain signal x_n is:

$$IDFT\{X_k\} = x_n = \frac{1}{K} \sum_{k=0}^{K-1} X_k (e^{\frac{j2\pi kn}{K}}) \tag{2}$$

- One advantage of using the DFT and the IDFT is that they can be efficiently calculated. Fast Fourier transform (FFT) and the inverse fast Fourier transform (IFFT) are efficient implementations of the DFT and the IDFT and have enabled many new applications in digital signal processing.
- Similarly, the convolution-multiplication property of the DFT states that if

$$y_n = x_n \circledast h_n \tag{3}$$

then

$$Y_f = X_f H_f \tag{4}$$

where $\circledast$ denotes circulation convolution. In other words, circular convolution of two signals in time is equivalent to multiplication of DFTs of two signals in frequency. y_n is the circular convolution of x_n and h_n and is operationally

$$y_n = \sum_{m=0}^{K-1} x_{n-m(modK)} h_m \tag{5}$$

- Circular convolution is used because the convolution-multiplication property of the DFT requires that x_n is periodic with the period K.
- Doing cyclic prefix makes the (K + g) data symbols look periodic, at least for the duration over which circular convolution is performed.
- In DFT, being able to perform circular convolution is what makes the relationship $Y_k = X_k \ H_k$ true.
- Therefore, at the OFDM receiver, a simple linear equalization can be used to recover the input (transmitted) X_k in the frequency domain by,

$$X_k = \frac{Y_k}{H_k} \tag{6}$$

In actuality, the channel also introduces noise n_k

$$Y_k = X_k H_k + n_k \tag{7}$$

and the equalized data symbol is:

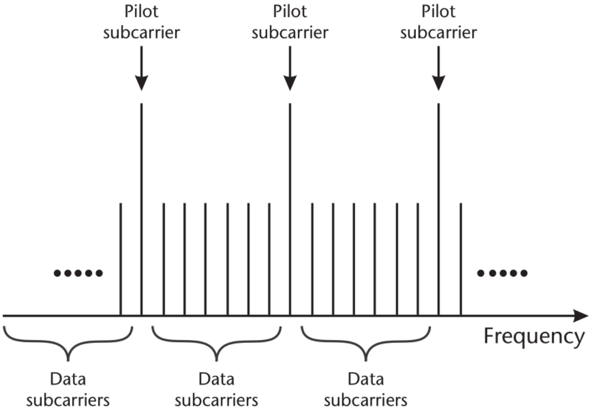
$$\frac{Y_k}{H_k} = \frac{X_k H_k}{H_k} + \frac{n_k}{H_k} = X_k + \frac{n_k}{H_k} \tag{8}$$

- In general, H_k(i.e., the channel) has to be known or at least estimated before the transmitted signal X_k can be recovered.
- Each subcarrier k experiences its own channel response H_k , and the channel response may be different at different frequencies.
- This means that each subcarrier k requires its own estimated channel response H_k .Therfore, we need K separate equalizers for K sub-carriers.

Pilot Sub carriers

- In OFDM, a number of the subcarriers are used as pilot subcarriers. Pilot sub-carriers carry known signals, and the receiver can estimate the response of the channel based on what are actually received on the pilot subcarriers.

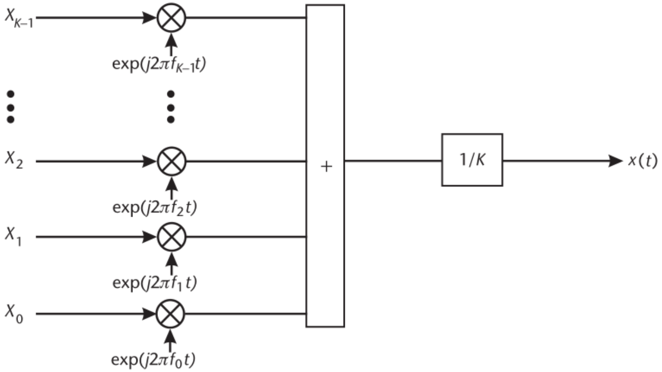
Pilots for estimating Channel Response



- Pilot sub-carrier - More Power - Channel Estimates reliable
- Because the channel response may be different at different frequencies, the actual response for a data subcarrier has to be interpolated based on measurements of the two nearest pilot subcarriers.

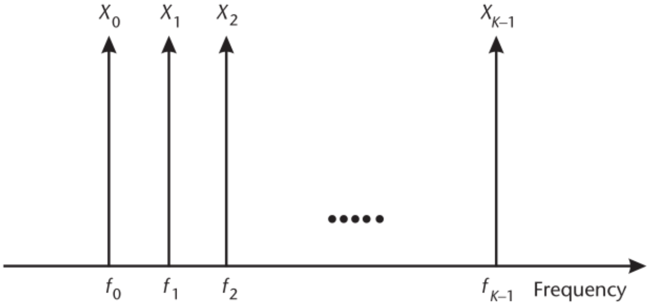
An OFDM symbol

- Thus far, we have stated that the OFDM spectrum (over the duration of an OFDM symbol) consists of a group of overlapping subcarriers.
- A multicarrier signal can be produced conventionally by a series of complex multipliers.



OFDM Symbol Contd..

- This series of complex multipliers generates a multicarrier signal x(t) that has the spectrum shown in Figure . The delta functions in frequency correspond to the complex sinusoids in time.



- The ensemble of complex sinusoids shown in the figure can be characterized by a series of complex data symbols X_k carried by a series of complex subcarriers $\exp(j2\pi fkt)$. In the complex baseband equivalent form, it is

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{j2\pi f_k t} \quad (9)$$

  where f_k is the center frequency of the kth subcarrier and K is the number of subcarriers.
- The spectrum of the multicarrier signal consists of delta functions scaled by complex data symbols at f_k .The spectral lines constitute the multicarrier signal because the complex sinusoids $\exp(j2\pi f_k t)$ exist for all time.

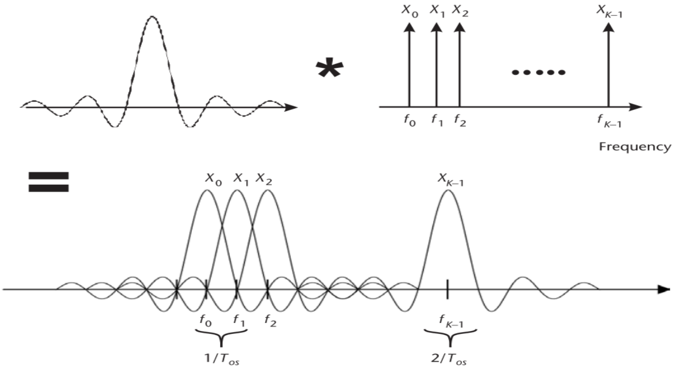
- Now, let us truncate the multicarrier signal in time so that it exists only for a limited duration T_{os} , that is,

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{j2\pi f_k t} \quad 0 < T < T_{os} \tag{10}$$

T_{os} is really the duration of an OFDM symbol.

- Limiting any signal to a range in time is equivalent to multiplying it by a rectangular function in time, and multiplication by a rectangular function in time is equivalent to convolution with a sinc function in frequency. If the rectangular function in time lasts T_{os} seconds, then its corresponding sinc function in frequency is $2/T_{os}$ wide (between the first two zeros).

- Therefore, truncating the multicarrier signal in time results in a magnitude spectrum that is the convolution of a series of delta functions with a sinc function. Figure shows that convolution of a series of delta functions with a sinc function results in copies of the sinc function duplicated at where the delta functions used to be.



Analog Version of the multicarrier signal

- If the sinc functions overlap, then according to Figure the difference between the centers of two adjacent sinc functions is $1/Tos$, so $f_k = k/T_{os}$. Substituting k/T_{os} for f_k in the Eq yields:

$$x(t) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kt}{T_{os}}} \quad 0 < T < T_{os} \tag{11}$$

- This equation is the continuous-time (analog) version of the multicarrier signal. In other words, it is the signal found the digital-to-analog converter.

Discrete Version of the multicarrier signal

- To derive the discrete-time (digital) form of the multicarrier signal, one proceeds to sample x(t) in time. Remember that x(t) exists only between t = 0 and t = T_{os} . In the duration of T_{os} seconds, K equally spaced samples are taken in time, so the nth sample takes place at t = $(T_{os}/K)n$. Replacing t with $(T_{os}/K)n$ produces:

$$x(\frac{T_{os}n}{K}) = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi k T_{os} n}{T_{os}K}} = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \qquad (12)$$

$$0 < T < T_{os}$$

which can be written as

$$x(n) = x_n = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \quad 0 < T < T_{os} \qquad (13)$$

because the argument of x(·) in discrete time is the sample number n itself

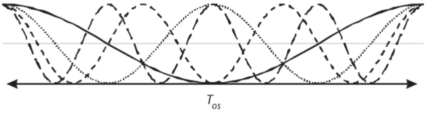
DFT Implementation

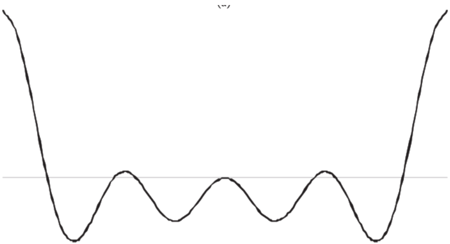
- Due to sampling, Eq.13, is the discrete-time (digital) form of the multicarrier signal. In other words, it is the signal found before the digital-to-analog converter .
- More importantly, one can easily recognize now that Eq.13 for x_n is simply the IDFT of X_k .What this means is that x_n in time (within an OFDM symbol) can be easily generated by a K-point IDFT function, that is,

$$IDFT\{X_k\} = x_n = \frac{1}{K}\sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kn}{K}} \quad 0 < T < T_{os} \tag{14}$$

- Having adjacent sync functions separate by $1/T_{os}$ peak-to-peak also enables the discrete-time version of the OFDM signal to match the IDFT of X_k. This match allows the generation of the OFDM signal using IDFT rather than using many complex multipliers.

OFDM Symbol in Time Domain





- The OFDM symbol lasts from t = 0 to t = T_{os} . In particular, OFDM symbol is made up of four data symbols (four subcarriers), and each subcarrier at a specific frequency is represented by a (truncated) cosinusoid with that frequency. The four subcarriers all have the same magnitude (e.g., 1, 1, 1, 1); thus, the four subcarriers all carry identical data symbols (e.g., 1, 1, 1, 1).

- Two important observations can be made regarding this figure. First, in an OFDM symbol, each data symbol lasts the entire T os . Second, these four subcarriers have frequencies $1/T_{os}$,$2/T_{os}$,$3/T_{os}$,$4/T_{os}$ (at baseband); thus:

  - The subcarrier with freq $1/T_{os}$ completes one cycle T_{os}
  - The subcarrier with freq $2/T_{os}$ completes two cycle T_{os}
  - The subcarrier with freq $3/T_{os}$ completes three cycle T_{os}
  - The subcarrier with freq $4/T_{os}$ completes four cycle T_{os}

- In other words, a subcarrier always completes an integer number of cycles from t = 0 to t = T_{os}.Figure depicts the actual superposition of four data symbols in time over the duration of T_{os} .
- It is easy to recognize orthogonality among subcarriers in the frequency domain.
- Because a data symbol X_k is recovered at the peak of the sync function, other sync functions do not interfere with X_k

Orthogonality in time domain?

- While it is straightforward to see orthogonality among subcarriers in the frequency domain, can one quantitatively show that the subcarriers (sync functions shifted by $1/T_{os}$) are orthogonal to each other and do not interfere with each other while in their analog form?
- To put it another way, can one be sure that the data symbols X_k carried by the subcarriers do not interfere with each other?
- The answer is yes, and such a proof can be more clearly shown in the time domain.

Orthogonality of SubCarriers

To demonstrate the orthogonality among subcarriers in the time domain, we multiply the analog signal x(t) by the complex conjugate of another subcarrier and integrate over the duration of an OFDM symbol ($0 < t < T_{os}$):

$$\int_0^{T_{os}} x(t) e^{\dfrac{-j2\pi lt}{T_{os}}} dt \;\; 0 < T < T_{os} \tag{15}$$

Note that the complex conjugate of this other subcarrier has an arbitrary frequency l/T_{os} . This integral is evaluated as follows:

Proof of Orthogonality

$$
\begin{aligned}
\int_0^{T_{os}} x(t) e^{\frac{-j2\pi lt}{T_{os}}} dt &= \frac{1}{K} \int_0^{T_{os}} e^{\frac{-j2\pi lt}{T_{os}}} \sum_{k=0}^{K-1} X_k e^{\frac{j2\pi kt}{T_{os}}} dt \\
&= \frac{1}{K} \sum_{k=0}^{K-1} X_k \int_0^{T_{os}} e^{\frac{-j2\pi lt}{T_{os}}} e^{\frac{j2\pi kt}{T_{os}}} dt \\
&= \frac{1}{K} \sum_{k=0}^{K-1} X_k \int_0^{T_{os}} e^{\frac{j2\pi (k-l)t}{T_{os}}} dt \\
&= \frac{T_{os} X_k}{K} \quad if\ l = k \\
&= 0 \qquad if\ l \neq q
\end{aligned} \tag{16}
$$

Observations

- If the complex conjugate of a subcarrier has the same center frequency l/Tos as the center frequency k/T_{os} of a subcarrier carrying X_k , then the data symbol X_k is recovered.
- Data symbols carried by other subcarriers l ($\neq k$) do not interfere with X_k. In other words, subcarrier k is orthogonal to any other subcarrier l ($\neq q$).
- The reason the above expression is 0 if $l \neq k$ is that a subcarrier (in an OFDM signal) always completes an integer number of cycles from t $=$ 0 to t $=$ T_{os} as shown in Figure 21, and the integration of a sinusoid over an integer number of cycles is always 0.

An Actual OFDM Transmitter

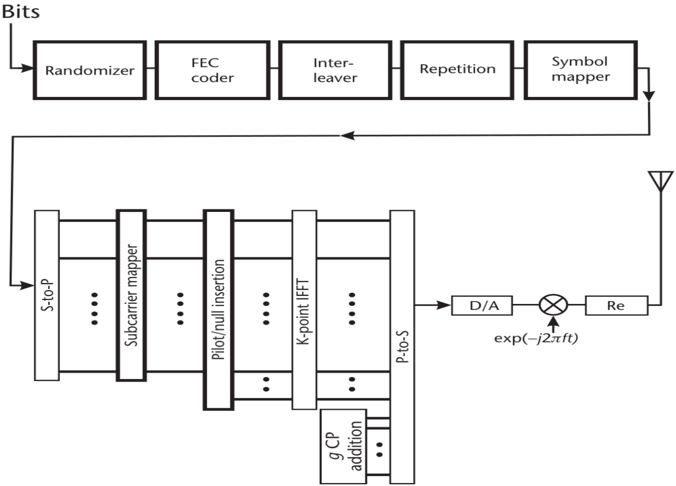


Figure: Basic structure of the OFDMA transmitter in IEEE 802.16e

Randomizer

The stream of information bits from the MAC layer are first fed into the data randomizer. The data randomizer XORs the data bits with bits produced by a shift register. The randomizer has three purposes:

- It scrambles the bits so that a casual eavesdropping receiver cannot easily intercept the data bits.
- It redistributes the bits to avoid long runs of 1s or 0s. A long run of 1s or 0s can cause a subcarrier to become unmodulated.
- It redistributes the bits to avoid long runs of 1s or 0s. A long run of 1s or 0s can cause the received bit stream (at the receiver) to lose synchronization. Bit-level synchronization requires a sufficient number of bit transitions (1-to-0 and 0-to-1) in a given time.
- The randomizer only operates on the information bits and is present in both the uplink and the downlink.

Forward Error Control (FEC)

- After data randomization, the scrambled bits go into the forward error correction (FEC) function, which uses an error-correcting code to add redundancy bits for error correction.
- In addition to convolutional codes , the IEEE 802.16e standard specifies three additional FEC codes: block turbo code, convolutional turbo code, and low density parity check code. The convolutional code is mandatory in IEEE 802.16e.

Interleaver

- After FEC, the coded bits are operated by the interleaver. The purpose of the interleaver is to ensure that the coded bits become sufficiently separated in frequency space and constellation space.
- In fact, the interleaver operates in two phases. The first phase is for frequency space. In the first phase, consecutive, coded bits are reordered to make sure that these bits are later mapped (by the subcarrier mapper) to nonadjacent subcarriers for frequency diversity.
- The second phase is for constellation space. In the second phase, consecutive, coded bits are reordered to make sure that these bits are later mapped fairly (by the symbol mapper) to more and less significant bits of the constellation.

Repetition

- After interleaving, the bits are organized into slots. At this point, the system can use the repetition function to further increase the reliability of the transmitted bits.
- The bits may be repeated by a repetition rate of 2, 4, or 6. Repetition provides a quick way for system designers to trade capacity for coverage.
- If a system designer would like to increase coverage, he or she can do so by increasing the repetition rate by a desired amount. This is because repetition decreases the required SNR at the receiver.
- However, the tradeoff is reduced capacity because available slots are occupied by repetitive bit.

Symbol Mapper

- The symbol mapper maps the (interleaved and/or repetitive) bits to data symbols based on the constellation used at the time (i.e., QPSK, 16-QAM, or 64-QAM).
- Note that each data symbol (later carried by a corresponding subcarrier) can be a QPSK, 16-QAM, or 64-QAM data symbol depending on the channel condition experienced by that subcarrier.
- If that subcarrier is experiencing high SINR, then it may carry a 64QAM data symbol to maximize bit rate.
- If that subcarrier is experiencing low SINR, then it may just carry a QPSK data symbol to ensure reliability.

SubCarrier Mappper

- The serial-to-parallel converter converts the serial stream of data symbols into parallel streams.
- Then the data symbols go into the subcarrier mapper, which assigns the individual data symbols to the individual subcarriers (i.e., assigning a subcarrier index to each data symbol).
- The subcarrier mapper is necessary in OFDMA because different data symbols may have come from different users, and assigning data symbols to different subcarriers allows multiple users to access the air interface simultaneously.

Pilot or Null Insertion

- The pilot/null insertion function inserts pilot symbols and null symbols.
- The pilot subcarriers are for channel estimation, and the null subcarriers include the guard subcarriers and the DC subcarrier.

An Actual OFDM Receiver

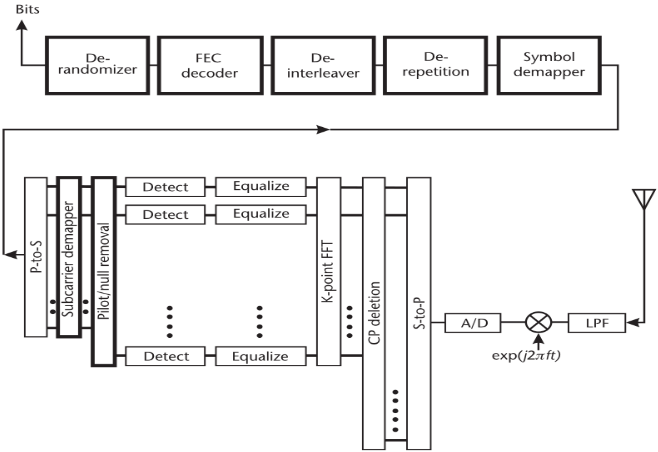


Figure: Basic structure of the OFDMA receiver complementing the transmitter in IEEE 802.16e.

Advantages of OFDM

Because OFDM also transmits using multiple narrowband subcarriers, it is robust against ISI and multipath fading, can adjust modulation and coding for each subcarrier, and has simple equalizers. In addition, OFDM offers two more important advantages:

- OFDM has low-complexity modulation; and
- OFDM achieves a better spectral efficiency than conventional FDM.

OFDMA

- Whereas OFDM assigns one block (in time) to one user, OFDMA is a method that assigns different groups of subcarriers (in frequency) to different users. This way, more than one user can access the air interface at the same time.
- Recall that in OFDM all K subcarriers are used to carry data for one user only. OFDM assigns all subcarriers to a single user at the same time, and only one user can transmit at a time.
- If multiple users want to transmit using OFDM, then those users have to take their turns in time.

OFDMA Transmitter

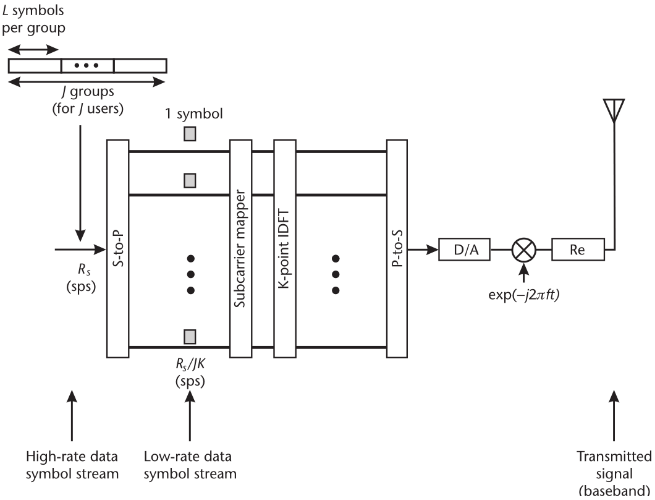
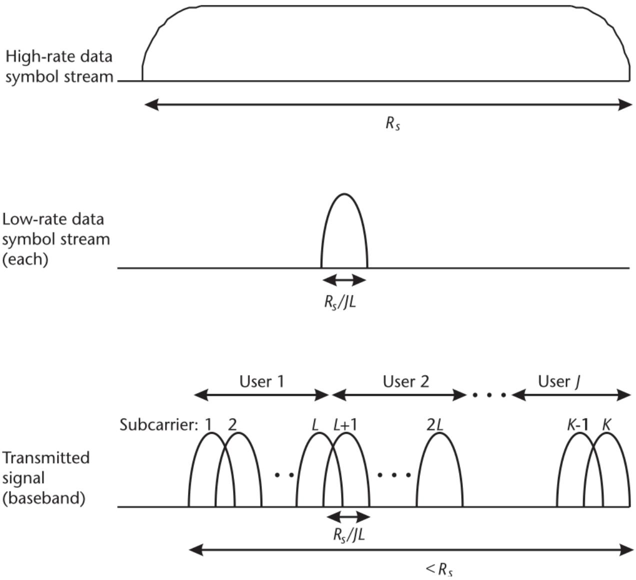


Figure: An OFDMA transmitter

- In OFDMA, instead of sequentially assigning OFDM symbols in time to different users, the system directly assigns subcarriers in frequency to different users.
- The high-rate stream of baseband data symbols is still running at a rate of $R_s sps$, and each data symbol lasts $1/R_s$ s. This high-rate stream consists of J groups of complex data symbols; each group contains L complex data symbols, and each group (of L data symbols) is later assigned to a different user. So there are a total of JL complex data symbols in J groups.
- The serial-to-parallel converter assigns the high-rate stream into JL separate low-rate substreams; each low-rate substream has a rate of R_s/JL sps. In doing so, the serial-to-parallel converter assigns successive data symbols (at its input) to JL separate low-rate substreams (at its outputs).
- So at any given time at the output of the serial-to-parallel converter, there is a set of JL data symbols in parallel.

- The subcarrier mapper maps JL data symbols to their respective subcarriers (which are assigned to different users). Specifically, the subcarrier mapper assigns J groups of data symbols to J users in frequency. In effect, the subcarrier mapper reorders the parallel data symbols according to the particular subcarriers assigned to each user.
- The set of mapped JL(= K) data symbols in parallel pass through the IDFT function, which transforms the K data symbols. The K transformed symbols in K substreams then pass through the parallel-to-serial converter that puts the K transformed symbols in series.
- This block of K transformed symbols in series constitutes a single OFDM symbol. Successive OFDM symbols at the output of the parallel-to-serial converter are running at a rate of R_s/K OFDM symbols per second, and each OFDM symbol lasts $K/R_s s$.


High-rate data symbol stream
R_s
Low-rate data symbol stream (each)
R_s/JL
Transmitted signal (baseband)
User 1
User 2
User J
Subcarrier: 1 2
L
L+1
2L
K-1
K
R_s/JL
$< R_s$

- Figure shows the spectrums of the high-rate stream, the low-rate sub-streams, and the transmitted signal. In particular, the spectrum of the transmitted signal is shown over J groups of data symbols, or one OFDM symbol, only.
- The subcarriers are spaced so that they overlap in frequency but are orthogonal because a data symbol is recovered at the peak of a subcarrier. Note that, in this case, data symbols belonging to a user are carried by contiguous subcarriers.

Assinging Users Data symbols to subcarriers

- In general, there are two ways to assign users' data symbols to subcarriers: distributed and contiguous.

Distributed subcarriers arrangement

- In a distributed subcarriers arrangement, subcarriers are assigned pseudorandomly to users.

Continous subcarriers arrangment

- In a contiguous subcarriers arrangement,subcarriers are assigned to users in continuous sets.

Multiple Users using OFDM

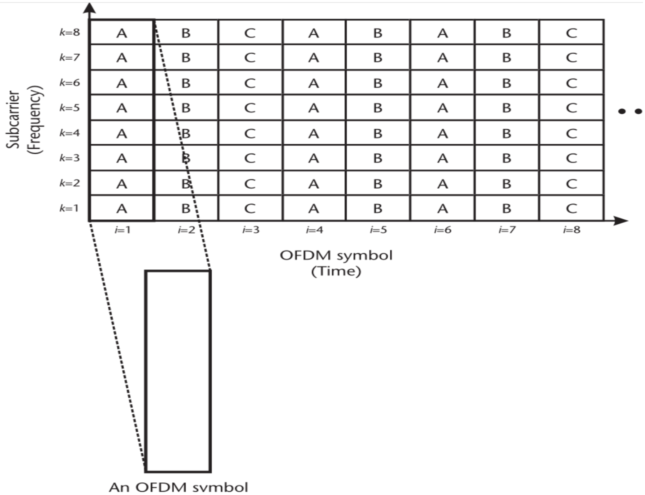


Figure: Multiple users using OFDM.

Multiple Users using OFDMA

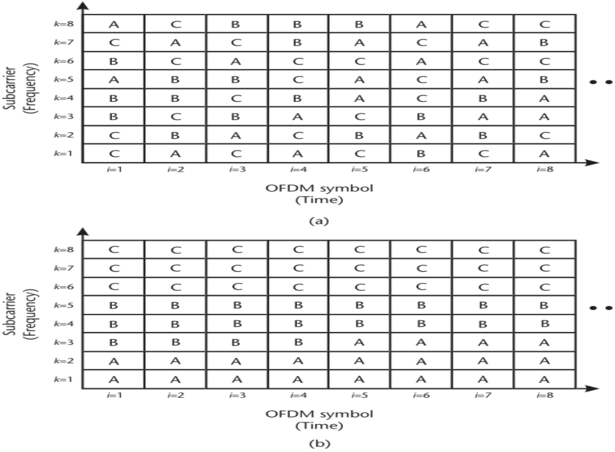


Figure: Multiple users using OFDMA: (a) distributed subcarriers and (b) contiguous subcarriers.

Distributed subcarriers arrangement

- Figure (a) illustrates the arrangement of distributed subcarriers, where users are assigned pseudorandomly to subcarriers.
- For example, in the first OFDM symbol, user A is assigned two subcarriers ($k = 5, 8$), whereas user B is assigned three subcarriers ($k = 3, 4, 6$).
- Distributing subcarriers pseudorandomly affords frequency diversity (to a single user)

Continuous subcarriers arrangement

- Figure (b) illustrates the arrangement of contiguous subcarriers.
- In contiguous subcarriers, subcarriers are assigned to users in continuous groups.
- For example, in the first OFDM symbol, user A is assigned two subcarriers (k = 1, 2),user B is assigned three subcarriers (k = 3, 4, 5), and user C is assigned three subcarriers (k = 6, 7, 8). Contiguous subcarriers can take advantage of multiuser diversity.

Frequency Diversity

- Frequency diversity is achieved by forming a subchannel through distributed subcarriers.
- As far as a single user is concerned, such a distribution of subcarriers offers frequency diversity. This is so because if a user's subcarriers are distributed pseudorandomly, some of its subcarriers likely would not experience fades while some of its other subcarriers would.
- Frequency diversity afforded by distributed subcarriers is well suited for mobile users because as a mobile user changes its location, the user experiences different multipath fadings at different locations.

Example for mobile user

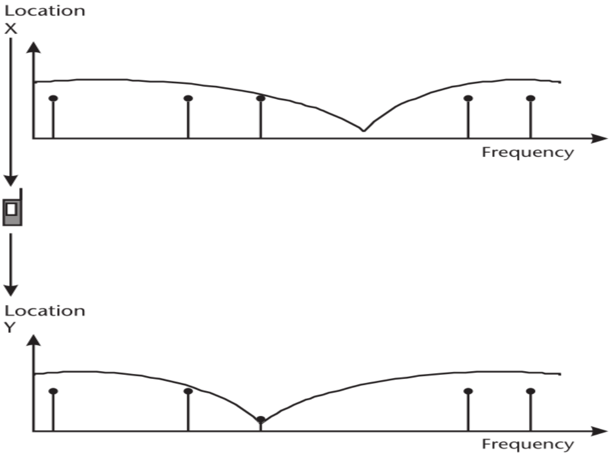


Figure: mobile user travels from location X to location Y. At location X, none of the user's subcarriers is degraded by the channel response. At location Y, a subcarrier is experiencing deep fade.

Multi User Diversity

- Multiuser diversity occurs when different users at different locations experience different channel responses.
- This form of diversity can be achieved by forming a subchannel through contiguous subcarriers. For contiguous subcarriers, a group of adjacent subcarriers are assigned to a single user.
- This scheme cannot take advantage of frequency diversity because all of the user's subcarriers are in the same vicinity of the spectrum. If a deep fade falls on top of those subcarriers belonging to a user, then that user will experience degraded channel. However, contiguous subcarriers can offer multiuser diversity.
- Multiuser diversity afforded by contiguous subcarriers is well suited for fixed users because a fixed user's location does not change, so the channel response experienced by the user is relatively constant. This way, the system can assign a user a set of contiguous subcarriers based on the user's channel response.

Example for fixed user

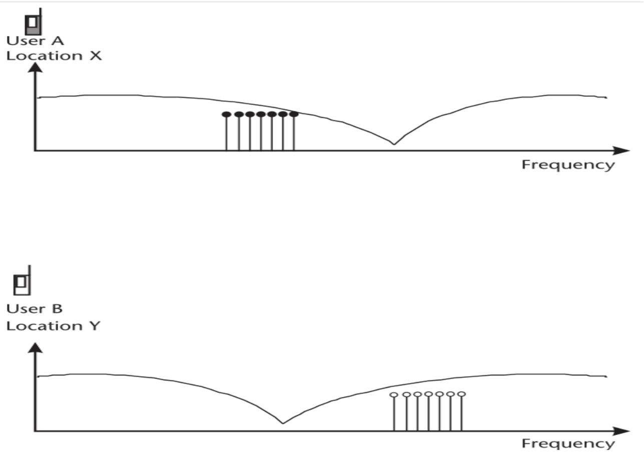


Figure: User A is at location X and user B is at location Y. At location X, user A's subcarriers are experiencing a good channel response (away from the null). At location Y, user B's subcarriers are also experiencing a good channel response (away from the null).

Saving Power!

- Typically, the base station measures the channel response across the band by using pilot subcarriers that locate throughout the band. In OFDMA systems, it is possible to assign subcarriers based on their SINRs. For contiguous carriers, the base station can assign to a user a set of contiguous subcarriers that experience high SINR.
- For a fixed user, contiguous subcarriers are especially helpful on the uplink in conserving the user's battery power. By transmitting on those subcarriers that experience strong SINR, the user device does not have to expend much power to attain a desired uplink bit rate.

Remarks

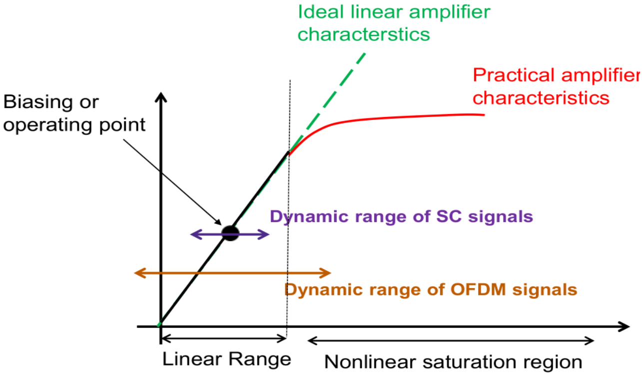
- In dynamically assigning subcarriers to users, the subcarrier mapper performs an important scheduler function, which "schedules" different subcarriers to carry different users' data symbols.
- The change in granted bandwidth as a function of time is probably due to different users' bandwidth requests at different times subject to any quality-of-service (QoS) constraints.
- In fact, the scheduler makes optimizations decisions on the assignments of subcarriers and allocation of bandwidth resources based on multiple factors, including a user's current request for bandwidth, other users' pending (and competing) requests for bandwidth, users' QoS requirements, and channel quality experienced by each user.

OFDMA Physical Layer Applications

- Because OFDMA can dynamically allocate resources both in frequency (subcarriers) and in time (OFDM symbols), it is expected that broadband mobile systems will mostly use the more flexible OFDMA in the physical layer.
- In fact, the Mobile WiMAX System Profile specifies only the OFDMA implementation at the physical layer because OFDMA's scalable architecture is more suitable for mobile usage.

Issues of OFDM & and OFDMA

- Frequency synchronization
- Dynamic Range of OFDM symbols (see figure 31). Amplifiers with high linear-range are costly.Operating in non-linear region causes distorion.



References

- Samuel C. Yang OFDMA System Analysis and Design
- R. Van Nee, and R. Prasad, OFDM for Multimedia Communications, Artech House, 2000.
- Andrea Goldsmith Book Chapter 12 Multicarrier Communications
- Behrouz A. Forouzan Data Communications and Networking
- U.Madhow,Introduction to Communication systems,2014
- Litwin, L, OFDM: An Introduction to Multicarrier Modulation," IEEE Potential, April/ May 2000

Thank you